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Competent Condition For Geometric Properties Q-Starlikeness And Q-Convexity Of Mittag-Leffler Function

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Abstract: This paper focus on the competent conditions for q-starlikeness and q-convexity of the Mittag-Leffler function. The Mittag-Leffler function, a generalization of the exponential function, plays a critical role in fractional calculus and differential equations. This work examines the function within the framework of q-calculus, an extension of classical calculus that introduces a deformation parameter q.
We derive sufficient conditions under which the Mittag-Leffler function exhibits q-starlikeness and q-convexity, properties that are pivotal for ensuring the function's regularity and univalence in specific domains. Utilizing techniques from geometric function theory, we establish criteria that involve the parameters of the Mittag-Leffler function and the deformation parameter q. Our results are found in terms of Fox Wright function which provide a comprehensive understanding of how these parameters influence the geometric behavior of the function, thereby contributing to the broader theory of q-analytic functions. This investigation not only extends the existing theory of the Mittag-Leffler function but also opens new pathways for its application in mathematical and physical problems characterized by q-deformations.

Keywords and Phrases: Univalent function, q-derivative operator, q-starlike function, q-convex function, Mittag Leffler function.

MSC: 30C45, 30C50, 30C80

Statements and Declarations

Competing Interests: The authors declare that there are no conflicts of interest regarding the publication of this paper.

1. Introduction and Preliminaries

Geometric function theory and q-calculus are attractive topics in study due to their numerous applications in engineering fields including fluid mechanics, mathematical biology, and chemistry, as well as in pure and applied mathematics. From last some years geometric properties like q-starlikeness, q-convexity and univalent, bi-valent function are discussed by many researchers and many new results are welcomed in fractional calculus. The q-calculus (calculus without limit) was firstly introduced by Jackson [1,2] in terms of different operators of differential and integrals, that is analytic over a unit disc. Properties like q-starlikeness and q-convexity are studied by many scholars. In terms, Rahman et al. [3,4]studied certain subclasses of analytic functions involving higher-order q-derivative operators and in terms of Janowski function. Long P.et al.[5] researched on functional inequalities for several classes of q-starlikeness and q-convexity for analytic and multivalent function. Recently, Gour M.M.et al.[6] have shown the result on coefficient inequalities for the classes of q starlike and q-convex function. In addition, Srivastav et al.[7,8,9] published the work on geometric properties related to Janowski function. Further, Nandini et al.[10] also worked on coefficient inequalities for the classes of q starlike and q-convex function of reciprocal order. Also, Srivastav et al.[11] contributed a lot in terms of research on geometric function theory. In the articles [17,18,19] work have been done on Starlikeness and Convexity of Generalized Struve Functions,Bessel Function and other functions.
Suppose H is the class of all analytic functions in open unit disc $\mathfrak{D}$, where $\mathfrak{D} = \{ z \ ||z| < 1\}$
And Let $F \in \mathcal{H}$ in the form of $\mathcal{F}(z) = z + \sum_{n\geq 2} a_n z^n \quad \text{For } z \in \mathfrak{D}$
It is obvious that here$F(0) = 0$ and $\mathcal{F}'(0)$
It is the condition for the normalzation.

Definition1. 1. Now if $F \in \mathcal{H}$ and satisfy $R\left\{\frac{z\,\partial_q \mathcal{F}(z)}{\mathcal{F}(z)}\right\} > \alpha,$ for all $z \in \mathfrak{D}$.
Then $F \in \mathcal{S}_q^{\,*}(\alpha)$.Means F is a q- starlike function of order α.

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- If q=1 then $R\left\{\frac{z\mathcal{F}'(z)}{\mathcal{F}(z)}\right\} > \alpha$ for all $z \in \mathfrak{D}$

Then $F \in \mathcal{S}^*(\alpha)$ andF is a starlike function of order α

- If α=1 then $R\left\{\frac{z\mathcal{F}'(z)}{\mathcal{F}(z)}\right\} > 0$ for all $z \in \mathfrak{D}$

Then $F \in \mathcal{S}^*$ orF is a starlike function

Definition 1.2. Now if $F \in \mathcal{H}$ and satisfy $R\left\{\frac{\partial_q(z\,\partial_q\mathcal{F}(z))}{\partial_q\mathcal{F}(z)}\right\} > \alpha$, for all $z \in \mathfrak{D}$

Then $F \in \mathcal{C}_q{}^*(\alpha)$ or F is a q- Convex function of order α.

- If q=1 then $R\left\{\frac{\partial(z\,\partial\mathcal{F}(z))}{\partial\mathcal{F}(z)}\right\} > \alpha$ for all $z \in \mathfrak{D}$

Then $F \in \mathcal{C}^*(\alpha)$ it meansF is a Convex function of order α

- If α=1 then $R\left\{\frac{\partial(z\mathcal{F}'(z))}{\partial\mathcal{F}(z)}\right\} > 0$ for all $z \in \mathfrak{D}$

Then $F \in \mathcal{C}^*$ and F is a convex function

Definition 1.3. In q-calculus, differentials of functions are defined as
$d_q\mathcal{F}(x) = \mathcal{F}(qx) - \mathcal{F}(x)$and derivatives of a function is defined as fractions by the q-derivative as

$$D_q\mathcal{F}(x) = \frac{d_q\mathcal{F}(x)}{d_q x} = \frac{\mathcal{F}(qx) - \mathcal{F}(x)}{qx - x}$$

The function $D_q(x)$ has the Maclaurin's series representation is

$$D_q\mathcal{F}(x) = 1 + \sum_{n=2}^{\infty}[n]_q a_n z^n \text{ for all } z \in \mathfrak{D}$$

Where $[n]_q = \begin{cases} \frac{1-q^n}{1-q}, & n \in \mathbb{C} \\ \sum_{r=0}^{n-1} q^r & \text{if } n \in \mathbb{N} = \{1,2,3,\ldots\} \end{cases}$

Definition1.4. [20] Let two functions g &h are analytic in $\mathfrak{D}$then by subordination propertyg $\prec h$ or $g(z) \prec h(z)$, and if there exists a Schwartz function$\mu(z)$analytic in $\mathfrak{D}$ and satisfy
$\mu(0) = 0, |\mu(z)| < 1$Such that $g(z) = h(\mu(z))$,
starlike and convex function in subordination form as

$\mathcal{S}^*(t) = \{\mathcal{F} \in \mathcal{H} : \left\{\frac{z\mathcal{F}'(z)}{\mathcal{F}(z)}\right\} \prec t(z)\}$ where $t(z) = \frac{1+z}{1-z}$

And $\mathcal{S}^*(A,B) = \{\mathcal{F} \in \mathcal{H} : \left\{\frac{z\mathcal{F}'(z)}{\mathcal{F}(z)}\right\} \prec \frac{1+Az}{1+Bz}\}$ where $-1 \le B < A \le 1$(1.4a)

- If A=1 , B =-1 then $\mathcal{S}^*(A,B) = \mathcal{S}^*$

Ismail et al. [21] studied and discovered a q-extension of the class $\mathcal{S}^*$ of starlike functions
In terms of Janowski function if f is an analytic function in open disc $\mathfrak{D}$ and

$$\text{if } \left|\frac{z\,\partial_q\mathcal{F}(z)}{\mathcal{F}(z)} - \frac{1}{1-q}\right| < \frac{1}{1-q}, z \in \mathfrak{D}$$

then $\frac{z\,\partial_q\mathcal{F}(z)}{\mathcal{F}(z)} \prec \frac{1+z}{1-qz}$,$z \in \mathfrak{D}$ Here $F \in \mathcal{S}_q{}^*$

- If q=1 then $F \in \mathcal{S}^*$

If$\frac{z\,\partial_q\mathcal{F}(z)}{\mathcal{F}(z)} \prec \frac{(A+1)t(z)+(A-1)}{(B+1)t(z)-(B-1)}$, $z \in \mathfrak{D}$$t(z) = \frac{1+z}{1-qz}$
then$\mathcal{F} \in \mathcal{S}_q{}^*[A,B]$

- If A= 1 and B=-1 then $F \in \mathcal{S}_q{}^*$

Similarly Janowski form in convex function by the subordination property

$\mathcal{C}^*(t) = \{\mathcal{F} \in \mathcal{H} : \left\{\frac{\partial[z\mathcal{F}'(z)]}{\partial\mathcal{F}(z)}\right\} \prec t(z)\}$ where $t(z) = \frac{1+z}{1-z}$

And $\mathcal{C}^*(A,B) = \{\mathcal{F} \in \mathcal{H} : \left\{\frac{\partial[z\mathcal{F}'(z)]}{\partial\mathcal{F}(z)}\right\} \prec \frac{1+Az}{1+Bz}\}$ where $-1 \le B < A \le 1$

- If A=1 , B =-1 then $\mathcal{C}^*(A,B) = \mathcal{C}^*$

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$$\text{if } \left|\frac{\partial z\,\partial_q\mathcal{F}(z)}{\partial\mathcal{F}(z)} - \frac{1}{1-q}\right| < \frac{1}{1-q}, z \in \mathfrak{D}$$

then $\frac{\partial z\,\partial_q\mathcal{F}(z)}{\partial\mathcal{F}(z)} \prec \frac{1+z}{1-qz}$,$z \in \mathfrak{D}$ Here $F \in \mathcal{C}_q{}^*$

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- if q=1 then $F \in \mathcal{C}^*$

now If $\frac{\partial z\, \partial_q \mathcal{F}(z)}{\partial \mathcal{F}(z)} \prec \frac{(A+1)t(z)+(A-1)}{(B+1)t(z)-(B-1)}$, $z \in \mathfrak{D} t(z) = \frac{1+z}{1-qz}$ then $\mathcal{F} \in \mathcal{C}_q{}^*[A,B]$

- if A= 1 and B=-1 then $F \in \mathcal{C}_q{}^*$

In [11] Mittag Leffler function is defined as

$E_p(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(pk+1)}$ and $E_{p,q}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(pk+q)}$ (1)

Generalized form of Mittag Leffler function introduced by Al-Bassam and Luchko [22]

$E_{p,q}^{(m)}(z) = E_{(p_1,q_1)(p2,q2)(p3,q3)\ldots\ldots(pm,qm)} = \sum_{n=0}^{\infty} \frac{z^n}{\prod_{k=1}^{m} \Gamma(p_k n + q_k)}$ (2)

In terms of Fox -Wright Function Mittag Leffler function[11] is

$E_{p,q}^{(m)}(z) = \left[\begin{matrix} (1,1) & ; & \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & & z \end{matrix}\right]$

Where $[p_k, q_k, \in R^+ \ (k = 1,2,3,\ldots m);\ m \in N;\ z \in C\ ]$

Here we can see $E_p(z) = E_{p,1}^{(1)}(z)$ and $E_{p,q}(z) = E_{p,q}^{(1)}(z)$

So Normalized M-L function is

$\prod_{k=1}^{m} q_k\, z\, E_{p,q}^{(m)}(z) = \Phi(z) = z + \sum_{n=2}^{\infty} \frac{z^n}{\prod_{k=1}^{m} \Gamma(p_k(n-1)+q_k)}$(1)

here $\Phi(0)=0=\Phi'(z) - 1$

And by equating from $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$

We get

$$a_n = \frac{1}{\prod_{k=1}^{m} \Gamma(p_k(n-1)+q_k)}$$

In this research paper, we investigate some sufficient conditions of q – starlikeness and q-convexity for Mittag Leffler function by using sufficient conditions defined by Shrivastava[9].Some similar work have also done by Gour et al.[23,24] and sufficient condition of starlike function for multivalent function by Goyal et al. [25].

Lemma 1.1.[9]Suppose $z \in \mathcal{S}_q^*[A,B]$ if it is achieving below condition

$$\sum_{n=2}^{\infty} (2q[n-1]_q + |(B+1)[n]_q - (A+1)|)|a_n| < |B-A|$$

Lemma 1.2.[9]Suppose $z \in \mathcal{C}_q^*[A,B]$ if it is achieving below condition

$\sum_{n=2}^{\infty} [n]_q (2q[n-1]_q + |(B+1)[n]_q - (A+1)|)|a_n| < |B-A|$.

2. Main Results

Theorem 2.1. Let $L(\alpha,\beta;q)$ be defined as follows

$$L(\alpha,\beta:q) = \left(\frac{2q+(B+1)}{1-q} + (A+1)\right) {}_1\psi_m \left[\begin{matrix} (1,1) & ; & \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & & 1 \end{matrix}\right]$$
$$- \frac{(B+3)q}{1-q} ({}_1\psi_m \left[\begin{matrix} (1,1) & ; & \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & & q \end{matrix}\right] + \frac{1}{\prod_{k=1}^{m} q_k}(A+B+2)$$

If the inequality $L(\alpha,\beta:q) < |B-A|$

holds, then function $\Phi(z) = \prod_{k=1}^{m} q_k\, z\, E_{p,q}^{(m)}(z) \in \mathcal{S}_q^*[A,B]$

Proof: Here

$$\prod_{k=1}^{m} q_k\, z\, E_{p,q}^{(m)}(z) = \Phi(z) = z + \sum_{n=2}^{\infty} \frac{z^n}{\prod_{k=1}^{m} \Gamma(p_k(n-1)+q_k)}$$

By comparing with Normalized function

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad , z \in \mathfrak{D}$$

So

$$a_n = \frac{1}{\prod_{k=1}^{m} \Gamma(p_k(n-1)+q_k)}$$

From Lemma (1.1), any function $z \in \mathcal{S}_q^*[A,B]$

fulfill (1.1), Then for $\Phi(z)$ it is sufficient to show that (2) holds,

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Where $a_n = \frac{1}{\prod_{k=1}^{m} \Gamma(p_k(n-1)+q_k)}$ & $[n]_q = \frac{1-q^n}{1-q}$

Here, by using triangle's inequality we get

$$\sum_{n=2}^{\infty}(2q[n-1]_q + |(B+1)[n]_q - (A+1)|)|a_n| \le \sum_{n=2}^{\infty}(2q\frac{1-q^{n-1}}{1-q}|a_n| + \sum_{n=2}^{\infty}(B+1)\frac{1-q^n}{1-q}|a_n| + \sum_{n=2}^{\infty}(A+1)\,|a_n|$$

$$= \sum_{n=2}^{\infty}\left(\frac{2q+(B+1)}{1-q} + (A+1)\right)|a_n| - \sum_{n=2}^{\infty}\frac{(B+3)q^n}{1-q}|a_n|$$

$$= \left(\frac{2q+(B+1)}{1-q} + (A+1)\right)\sum_{n=2}^{\infty}\frac{1}{\prod_{k=1}^{m}\Gamma(p_k(n-1)+q_k)} - \frac{(B+3)q}{1-q}\sum_{n=2}^{\infty}\frac{1}{\prod_{k=1}^{m}\Gamma(p_k(n-1)+q_k)}q^{n-1}$$

By applying Fox Wright function property above condition convert in

$$= \left(\frac{2q+(B+1)}{1-q} + (A+1)\right)\left({}_1\psi_m\left[\begin{matrix}(1,1) & ; \\ (q_1,p_1)(q_2,p_2)\dots\dots(q_m,p_m); & 1\end{matrix}\right] - \frac{1}{\prod_{k=1}^{m}q_k}\right) - \frac{(B+3)q}{1-q}\left({}_1\psi_m\left[\begin{matrix}(1,1) & ; \\ (q_1,p_1)(q_2,p_2)\dots\dots(q_m,p_m); & q\end{matrix}\right] - \frac{1}{\prod_{k=1}^{m}q_k}\right)$$

$$= \left(\frac{2q+(B+1)}{1-q} + (A+1)\right){}_1\psi_m\left[\begin{matrix}(1,1) & ; \\ (q_1,p_1)(q_2,p_2)\dots\dots(q_m,p_m); & 1\end{matrix}\right] - \frac{(B+3)q}{1-q}{}_1\psi_m\left[\begin{matrix}(1,1) & ; \\ (q_1,p_1)(q_2,p_2)\dots\dots(q_m,p_m); & q\end{matrix}\right] + \frac{1}{\prod_{k=1}^{m}q_k}(A+B+2) \qquad (3)$$

$$= L(\alpha,\beta:q)$$

which conclude that the function $\Phi(z) = \prod_{k=1}^{m} q_k\, z\, E_{p,q}^{(m)}(z) \in \mathcal{S}_q^*[A,B]$

Corollary 2.1. Let A = z, B = 1 then above condition become

$$L^*(\alpha,\beta:q) = \left(\frac{2q+1}{1-q} + z + 1\right){}_1\psi_m\left[\begin{matrix}(1,1) & ; \\ (q_1,p_1)(q_2,p_2)\dots\dots(q_m,p_m); & 1\end{matrix}\right] - \frac{4q}{1-q}{}_1\psi_m\left[\begin{matrix}(1,1) & ; \\ (q_1,p_1)(q_2,p_2)\dots\dots(q_m,p_m); & q\end{matrix}\right] + \frac{1}{\prod_{k=1}^{m}q_k}(z+3)$$

If the inequality $L(\alpha,\beta:q) < 1 - z$, *holds*

then the function $\Phi(z) = \prod_{k=1}^{m} q_k\, z\, E_{p,q}^{(m)}(z) \in \mathcal{S}_q^*[z]$

- If z=0 then from above

$$L^*(\alpha,\beta:q) = \left(\frac{q+2}{1-q}\right){}_1\psi_m\left[\begin{matrix}(1,1) & ; \\ (q_1,p_1)(q_2,p_2)\dots\dots(q_m,p_m); & 1\end{matrix}\right] - \frac{4q}{1-q}{}_1\psi_m\left[\begin{matrix}(1,1) & ; \\ (q_1,p_1)(q_2,p_2)\dots\dots(q_m,p_m); & q\end{matrix}\right] + \frac{1}{\prod_{k=1}^{m}q_k}$$

If the inequality $L(\alpha,\beta:q) < 1$, *holds*

then the function $\Phi(z) = \prod_{k=1}^{m} q_k\, z\, E_{p,q}^{(m)}(z) \in \mathcal{S}_q^*[0]$

Theorem2.2. Let $M(\alpha,\beta:q)$, be defined as follows

$$M(\alpha,\beta:q) = \frac{1}{(1-q)^2}[(q+B+2+A(1-q)\left[{}_1\psi_m\left[\begin{matrix}(1,1) & ; \\ (q_1,p_1)(q_2,p_2)\dots\dots(q_m,p_m); & 1\end{matrix}\right]\right] - (Aq(1-q)+2Bq+q^2+5q)\left[{}_1\psi_m\left[\begin{matrix}(1,1) & ; \\ (q_1,p_1)(q_2,p_2)\dots\dots(q_m,p_m); & q\end{matrix}\right]\right] + (B+3)q^2\left[{}_1\psi_m\left[\begin{matrix}(1,1) & ; \\ (q_1,p_1)(q_2,p_2)\dots\dots(q_m,p_m); & q^2\end{matrix}\right]\right] + \frac{1}{\prod_{k=1}^{m}q_k}(A+B+2) \qquad (4)$$

If the inequality $M(\alpha,\beta:q) < |B-A|$

Holds, then function $\Phi(z) = \prod_{k=1}^{m} q_k\, z\, E_{p,q}^{(m)}(z) \in \mathcal{C}_q^*[A,B]$

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Proof: Here

$$\prod_{k=1}^{m} q_k\, z\, E_{p,q}^{(m)}(z) = \Phi(z) \;=\; z \;+ \sum_{n=2}^{\infty} \frac{z^n}{\prod_{k=1}^{m} \Gamma(p_k(n-1)+q_k)}$$

By comparing with Normalized function

$$= \; z + \sum_{n=2}^{\infty} a_n\, z^n \quad , z \in \mathfrak{D}$$

Where $a_n = \frac{1}{\prod_{k=1}^{m} \Gamma(p_k(n-1)+q_k)}$

From Lemma 1.2, any function $z \in \mathcal{C}_q^*[A,B]$ fulfills Lemma Then, for$\Phi(z)$ it is sufficient to show that equation (4) holds, for

$$a_n = \frac{1}{\prod_{k=1}^{m} \Gamma(p_k(n-1)+q_k)} \& [n]_q = \frac{1-q^n}{1-q}$$

As, by using triangle's inequality we get

$$\sum_{n=2}^{\infty} [n]_q (2q[n-1]_q + \left|(B+1)[n]_q - (A+1)\right|)|a_n|$$

$$\leq \sum_{n=2}^{\infty} (2q[n]_q \frac{1-q^{n-1}}{1-q}|a_n| + \sum_{n=2}^{\infty} (B+1)[n]_q \frac{1-q^n}{1-q}|a_n| + \sum_{n=2}^{\infty} (A+1)[q]_n\, |a_n|$$

$$= \quad \sum_{n=2}^{\infty} (2q \frac{1-q^n}{1-q} \frac{1-q^{n-1}}{1-q}|a_n| + \sum_{n=2}^{\infty} (B+1) \frac{1-q^n}{1-q} \frac{1-q^n}{1-q}|a_n| + \sum_{n=2}^{\infty} (A+1) \frac{1-q^n}{1-q}|a_n|$$

$$= \quad \sum_{n=2}^{\infty} \frac{2q+(B+1)+(A+1)(1-q)}{(1-q)^2}|a_n| + \sum_{n=2}^{\infty} \frac{2+(B+1)}{(1-q)^2} q^{2n}|a_n| - \sum_{n=2}^{\infty} \frac{(A+1)(1-q)+2(B+1)+2q+2}{(1-q)^2} q^n |a_n|$$

on applying Fox Wright function above equation becomes

$$= \frac{q+B+2+A(1-q)}{((1-q)^2} \left[{}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & 1 \end{matrix} \right] - \frac{1}{\prod_{k=1}^{m} q_k} \right] - \frac{Aq(1-q)+2Bq+q^2+5q}{(1-q)^2} \left[{}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & q \end{matrix} \right] - \frac{1}{\prod_{k=1}^{m} q_k} \right] + \frac{(B+1)q^2}{(1-q)^2} \left[{}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & q^2 \end{matrix} \right] - \frac{1}{\prod_{k=1}^{m} q_k} \right]$$

$$= \frac{1}{(1-q)^2} [(q+B+2+A(1-q) \left[{}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & 1 \end{matrix} \right] \right] - (Aq(1-q)+2Bq+q^2+5q) \left[{}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & q \end{matrix} \right] \right] + (B+3)q^2 \left[{}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & q^2 \end{matrix} \right] \right] + \frac{1}{\prod_{k=1}^{m} q_k} (A+B+2) \qquad (5)$$

$$= M(\alpha,\beta:q) < |B-A|$$

Therefore, the theorem's assumption implies Lemma (1.b),
hence function $\Phi(z) = \prod_{k=1}^{m} q_k\, z\, E_{p,q}^{(m)}(z) \in \mathcal{C}_q^*[A,B]$

Corollary 2.1. Let A= z, B = 1 then above condition become

$$M^*(\alpha,\beta:q) = \frac{1}{(1-q)^2} [(q+3+z(1-q) \left({}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & q^2 \end{matrix} \right] \right) - (zq(1-q)+2q+q^2+5q) \left({}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & q^2 \end{matrix} \right] \right) + 4q^2 \left({}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & q^2 \end{matrix} \right] \right) + \frac{1}{\prod_{k=1}^{m} q_k} (z+3) \qquad (6)$$

If the inequality$M^*(\alpha, \beta: q) < 1 - z$, holds

then the function$\Phi(z) = \prod_{k=1}^{m} q_k\, z\, E_{p,q}^{(m)}(z) \in \mathcal{C}_q^*[z]$

- If $z = 0$ then from (6)

$$M_1{}^*(\alpha,\beta:q) = \frac{1}{(1-q)^2} [(q+3+z) \left[{}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & 1 \end{matrix} \right] \right] - (2q+q^2+5q) \left[{}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & q \end{matrix} \right] \right] + 4q^2 \left[{}_1\psi_m \left[\begin{matrix} (1,1) & ; \\ (q_1,p_1)(q_2,p_2)\ldots\ldots(q_m,p_m); & q^2 \end{matrix} \right] \right] + 3\frac{1}{\prod_{k=1}^{m} q_k}$$

If the inequality $M_1^{*}(\alpha, \beta: q) < 1$, holds,

then the function $$\Phi(z) = \prod_{k=1}^{m} q_k \, z \, E_{p,q}^{(m)}(z) \in \mathcal{C}_q^{*}[0]$$

3. Conclusion

In this paper, the authors have derived a sufficient condition for Mittag -Leffler function resulting as a form of the Fox-Wright function. The Fox-Wright function is a generalized form of the hypergeometric function, often used in advanced mathematical and engineering problems due to its flexibility and encompassing nature and the Mittag-Leffler function is a significant function in fractional calculus and various applications in physics and engineering. The normalized version of this function, which typically ensures that it satisfies certain initial conditions, is used as the basis for the study. Researchers can use different function and get many more results in future.

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