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Edge Harmonic Mean Cordial Labeling of Some Cycle Related Graphs

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ABSTRACT
All the graphs considered in this article are simple and undirected. Let $G = (V(G), E(G))$ be a simple undirected Graph. A function $f: E(G) \rightarrow \{1,2\}$ is called Edge Harmonic Mean Cordial if the induced function $f^*: V(G) \rightarrow \{1,2\}$ defined by $f^*(x = uv) = \left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ or $f^*(x = uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ where u, v are the edges incident with the vertex x which satisfies the condition $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for any $i, j \in \{1,2\}$, where $v_f(x)$ and $e_f(x)$ denotes the number of vertices and number of edges with label x respectively. A Graph G is called Edge Harmonic Mean Cordial (EHMC) graph if it admits Edge Harmonic Mean Cordial labeling. In this article, we have discussed Edge Harmonic Mean Cordial Labeling of Some Cycle Related Graphs.

Keywords: Edge Harmonic Mean Cordial Labeling, Cycle, Complete Graph, Helm, Wheel.

AMS Subject Classification: 05C78, 05C76

Introduction

The notion of graph labeling in graph theory has garnered significant attention from scholars because of its wide-ranging and rigorous applications in domains such as communication network design and analysis, military surveillance, social sciences, optimization, and linear algebra. Various graph labelings are documented in the current body of literature. A dynamic survey of graph labeling by Gallian [2] is a condensed compilation of a lengthy bibliography of articles on the subject.

In this paper we introduce the Edge Harmonic Mean Cordial Labeling and investigate Edge Harmonic Mean Cordial Labeling of Some Cycle Related Graphs.

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$. For terminology and notation not defined here we follow Balakrishnan and Ranganathan [1].

Let $G = (V(G), E(G))$ be a simple undirected Graph. A function $f: E(G) \rightarrow \{1,2\}$ is called Edge Harmonic Mean Cordial if the induced function $f^*: V(G) \rightarrow \{1,2\}$ defined by $f^*(x = uv) = \left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ where, u, v are the edges incident with the vertex x which satisfies the condition $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for any $i, j \in \{1,2\}$, where $v_f(x)$ and $e_f(x)$ denotes the number of vertices and number of edges with label x respectively. A Graph G is called Edge Harmonic Mean Cordial (EHMC) graph if it admits Edge Harmonic Mean Cordial labeling. For the sake of convenience of the reader we use `EHMC' for edge harmonic mean cordial labeling.

Motivated by the interesting results proved in [3, 4, 5, 6, 8, 9] and on Root Cube Mean Cordial Labeling in [7, 10].

Definition 1.1. [1] Cycle is a closed trail in which all the vertices are distinct. It is denoted by C_n.

Definition 1.2. [1] The Helm graph H_n is the graph obtained from a wheel W_n by attaching a pendent edge at each vertex of the n cycle.

Definition 1.3. [2] A Closed Helm graph is the graph obtained from a Helm by joining each pendent vertex to form a cycle. It is denoted by CH_n.

Definition 1.4. [1] The Wheel graph W_n is a join of two graphs K_1 and C_n. *i.e.* $W_n = K_1 \vee C_n$.

Definition 1.5. [2] A Web graph Wb_n is the graph obtained by joining the pendent vertices of a helm to form a cycle and then adding a single pendent edge to each vertex of this outer cycle.

Definition 1.6. [1] A Complete graph is a type of graph where every pair of distinct vertices is connected by a single edge. It is denoted by K_n.

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Definition 1.7. [2] The Crown $C_n \odot K_1$ is obtained by joining a pendant edge to each vertex of cycle C_n.

Definition 1.8. [2] The Armed Crown is a graph in which path P_2 is attached at each vertex of cycle C_n by an edge. It is denoted by ACr_n where n is the number of vertices in cycle C_n.

Definition 1.9. [2] The Flower Fl_n is the graph obtained from helm H_n by joining each pendant vertex to the apex of helm.

Definition 1.10. [2] The Gear graph G_n is obtained from wheel W_n by subdividing each of its rim edge.

Definition 1.10. [2] A Shell graph is defined as a cycle C_n with $(n-3)$ chords sharing a common end point called the apex. It is denoted by S_n.

Main Results
Theorem 2.1. The Cycle C_n is EHMC if n is odd.
Proof. Let $G = (V, E) = C_n$ be the cycle. Note that $|V| = |E| = n$. Let $V = \{x_1, x_2, \dots, x_n\}$ be the vertex set and $E = \{e_1, e_2, \dots, e_n\}$ be the edge set of cycle C_n shown in the following figure - 1.

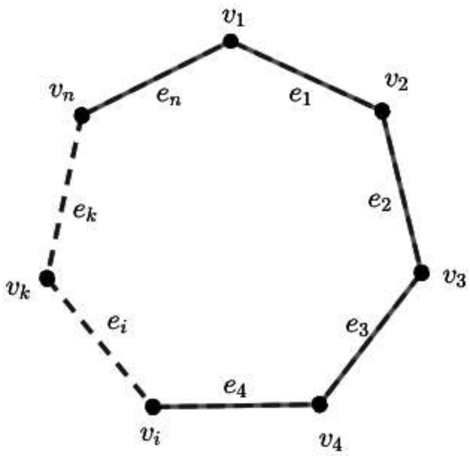

Figure – 1: C_n

Define a labeling function $f: E(C_n) \to \{1,2\}$ as follows,
$f(e_i) = 1$, if $1 \le i \le \frac{n-1}{2}$ and,
$f(e_i) = 2$, if $\frac{n+1}{2} \le i \le n$
Then $v_f(1) = \frac{n+1}{2}, v_f(2) = \frac{n-1}{2}$ and $e_f(1) = \frac{n-1}{2}, e_f(2) = \frac{n+1}{2}$. So, we have $|v_f(1) - v_f(2)| = 1$ and $|e_f(1) - e_f(2)| = 1$.
Hence, The Cycle C_n is EHMC if n is odd.

Illustration 2.2. EHMC labeling of C_7 is shown in following figure – 2.

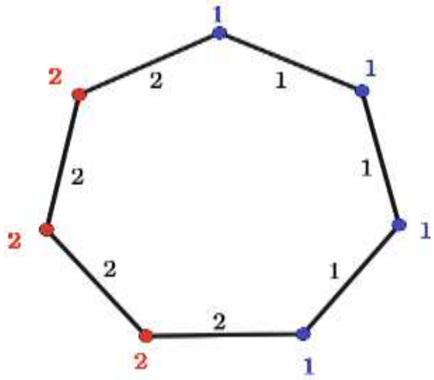

Figure – 2: C_7

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Theorem 2.3. The Cycle C_n is not EHMC if n is even.
Proof. Let $G=(V,E)=C_n$ be the cycle. Note that $|V|=|E|=n$. Let $V=\{x_1,x_2,\ldots,x_n\}$ be the vertex set and $E=\{e_1,e_2,\ldots,e_n\}$ be the edge set of cycle C_n.
If possible, let there be a EHMC labeling $f: E(C_n)\to\{1,2\}$ for graph C_n.
So, $e_f(1)=e_f(2)=\frac{n}{2}$. If we assign consecutive labeling 2 on $\frac{n}{2}$ edges of C_n, then we have $v_f(1)=\frac{n+2}{2}$ and $v_f(2)=\frac{n-2}{2}$.
Therefore $|v_f(1)-v_f(2)|=2>1$.
Without assuming consecutive labeling 2 on $\frac{n}{2}$ edges of C_n, we get $|v_f(1)-v_f(2)|>2$. Thus C_n does not satisfies EHMC labeling.
Hence, The Cycle C_n is not EHMC if n is even.

Theorem 2.4. The Helm H_n is EHMC.
Proof. Let $G=(V,E)=H_n$ be the helm graph. Note that $|V|=2n+1$ and $|E|=3n$. Let $V=\{v,v_1,v_2,\ldots,v_n,v_{n+1},v_{n+2},\ldots,v_{2n}\}$ be a vertex set and $E=\{e_1,e_2,\ldots,e_{3n}\}$ be a edge set of H_n shown in the following figure - 3.

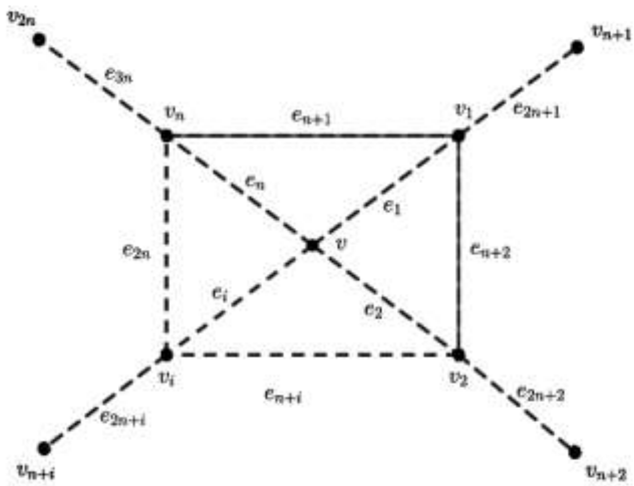

Figure – 3: H_n

Case 1: n is even
Define a labeling function $f: E(H_n)\to\{1,2\}$ as follows,
$f(e_i)=2$ if $1\le i\le\frac{n}{2}$,
$f(e_i)=2$ if $2n+1\le i\le 3n$,
$f(e_i)=1$ if $\frac{n}{2}+1\le i\le n$ and
$f(e_i)=1$ if $n+1\le i\le 2n$
Then $v_f(1)=n+1, v_f(2)=n$. So, we have $|v_f(1)-v_f(2)|=1$.

Case 2: n is odd
Define a labeling function $f: E(H_n)\to\{1,2\}$ as follows,
$f(e_i)=2$ if $1\le i\le\frac{n+1}{2}$,
$f(e_i)=2$ if $2n+1\le i\le 3n$,
$f(e_i)=1$ if $\frac{n+3}{2}\le i\le n$ and
$f(e_i)=1$ if $n+1\le i\le 2n$
Then $v_f(1)=n+1, v_f(2)=n$. So, we have $|v_f(1)-v_f(2)|=1$.
Hence, The Helm graph H_n is EHMC.

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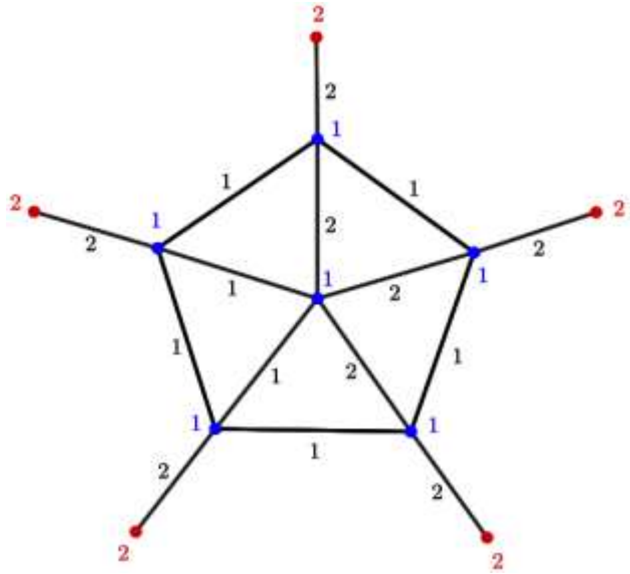
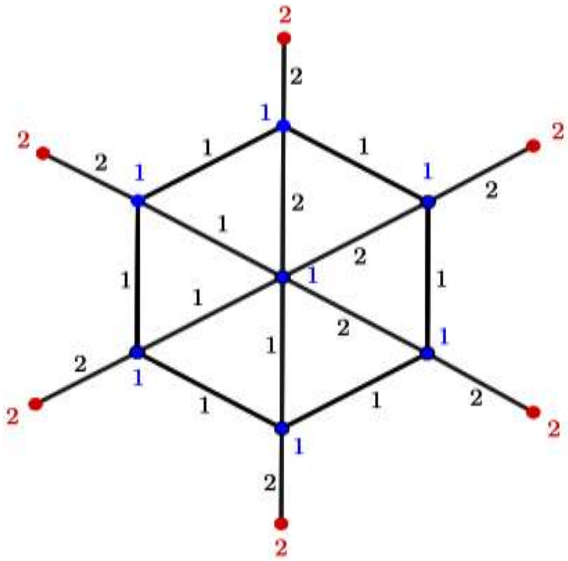
Illustration 2.5. EHMC labeling of H_5 and H_6 is shown in following figure - 4.



$\boldsymbol{H_5}$



$\boldsymbol{H_6}$

Figure – 4

Theorem 2.6. The Closed Helm graph CH_n is EHMC.
Proof. Let $G = (V, E) = CH_n$ be the Closed Helm graph. Note that $|V| = 2n + 1$ and $|E| = 4n$. Let $V = \{v, v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}, \ldots, v_{2n}\}$ the vertex set and $E = \{e_1, e_2, \ldots, e_n, e_{n+1}, \ldots, e_{2n}, e_{2n+1}, \ldots, e_{3n}, e_{3n+1}, \ldots, e_{4n}\}$ be the edge set of graph CH_n shown in the following figure - 5.

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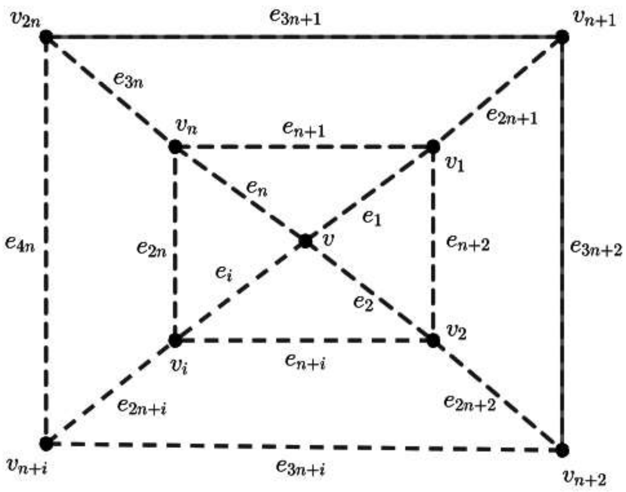
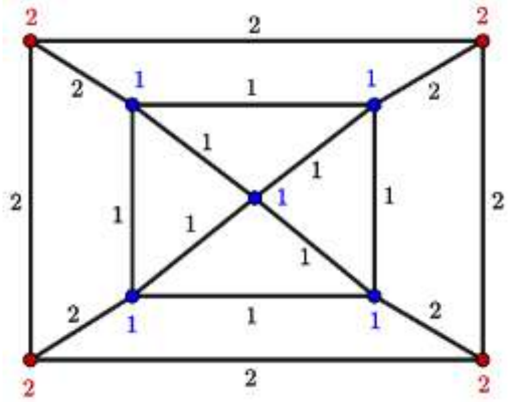


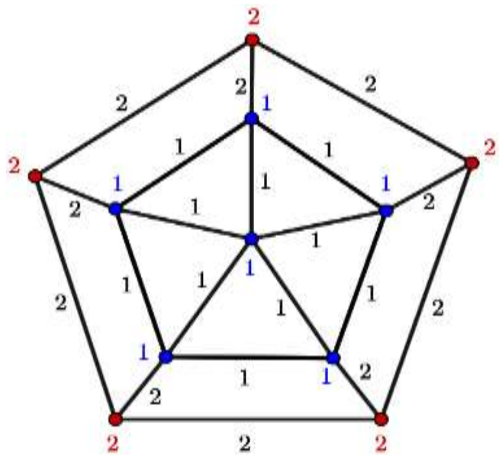
Figure – 5: $\boldsymbol{CH_n}$

Define a labeling function $f: E(CH_n) \rightarrow \{1,2\}$ as follows,
$f(e_i) = 1$ if $1 \leq i \leq 2n$ and
$f(e_i) = 2$ if $2n + 1 \leq i \leq 4n$.
Then $v_f(1) = n + 1, v_f(2) = n$. So, we have $\left|v_f(1) - v_f(2)\right| = 1$.
Hence, The Closed Helm graph CH_n is EHMC.

Illustration 2.5. EHMC labeling of CH_4 and CH_5 is shown in following figure - 6.



$\boldsymbol{CH_4}$



$\boldsymbol{CH_5}$
Figure – 6

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Theorem 2.6. The Wheel graph W_n is not EHMC for any n.
Proof. Let $G = (V, E) = W_n$ be the Wheel graph. Note that $|V| = n$ and $|E| = 2n - 2$. Let $V = \{v, v_1, v_2, \dots, v_{n-1}\}$ be the vertex set and $E = \{e_1, e_2, \dots, e_n, \dots, e_{2n-2}\}$ be the edge set of W_n.

Case 1: n is even
If possible, let there be a EHMC labeling $f: E(W_n) \rightarrow \{1,2\}$ for graph W_n.
If we assign consecutive labeling 2 on $\frac{2n-2}{2}$ edges of W_n, then we have $v_f(1) = \frac{n+4}{2}$ and $v_f(2) = \frac{n-2}{2}$. Therefore $|v_f(1) - v_f(2)| = 3 > 1$.
Without assuming consecutive labeling 2 on $\frac{2n-2}{2}$ edges of W_n, we get $|v_f(1) - v_f(2)| > 3$. Thus W_n does not satisfies EHMC labeling in this case.

Case 2: n is odd
If possible, let there be a EHMC labeling $f: E(W_n) \rightarrow \{1,2\}$ for graph W_n.
If we assign consecutive labeling 2 on $\frac{2n-2}{2}$ edges of W_n, then we have $v_f(1) = \frac{n+3}{2}$ and $v_f(2) = \frac{n-1}{2}$. Therefore $|v_f(1) - v_f(2)| = 2 > 1$.
Without assuming consecutive labeling 2 on $\frac{2n-2}{2}$ edges of W_n, we get $|v_f(1) - v_f(2)| > 2$. Thus W_n does not satisfies EHMC labeling in this case.
Hence, The Wheel W_n is not EHMC for any n.

Theorem 2.7. The Web graph Wb_n is EHMC.
Proof. Let $G = (V, E) = Wb_n$ be the Web graph. Note that $|V| = 3n + 1$ and $|E| = 5n$. Let $V = \{v, v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{2n}, v_{2n+1}, v_{2n+2}, \dots, v_{3n}\}$ be a vertex set and $E = \{e_1, e_2, \dots, e_{5n}\}$ be a edge set of Wb_n shown in the following figure - 7.

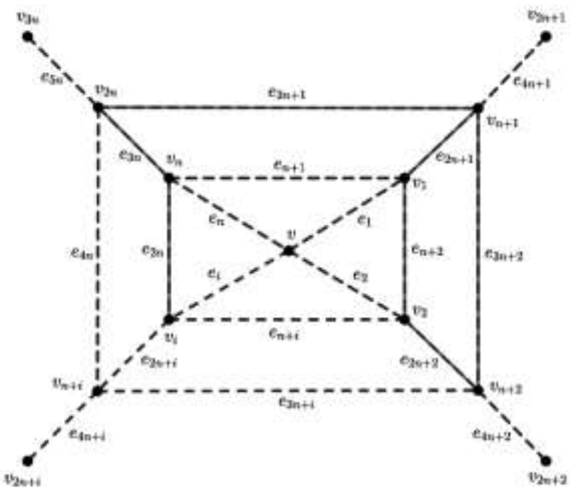

Figure – 7: Wb_n

Case 1: $5n$ is even
Define a labeling function $f: E(H_n) \rightarrow \{1,2\}$ as follows,
$f(e_i) = 1$ if $1 \leq i \leq \frac{5n}{2}$ and
$f(e_i) = 2$, if $\frac{5n+2}{2} \leq i \leq 5n$
Then $v_f(1) = \frac{3n+2}{2}, v_f(2) = \frac{3n}{2}$. So, we have $|v_f(1) - v_f(2)| = 1$.

Case 2: $5n$ is odd
Define a labeling function $f: E(H_n) \rightarrow \{1,2\}$ as follows,
$f(e_i) = 1$ if $1 \leq i \leq \frac{5n+1}{2}$ and
$f(e_i) = 2$,if $\frac{5n+1}{2} \leq i \leq 5n$
Then $v_f(1) = v_f(2) = \frac{3n+1}{2}$. So, we have $|v_f(1) - v_f(2)| = 0$.
Hence, The Web graph Wb_n is HMC.

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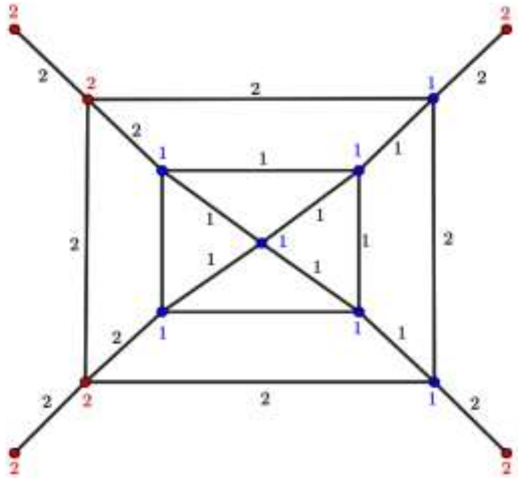
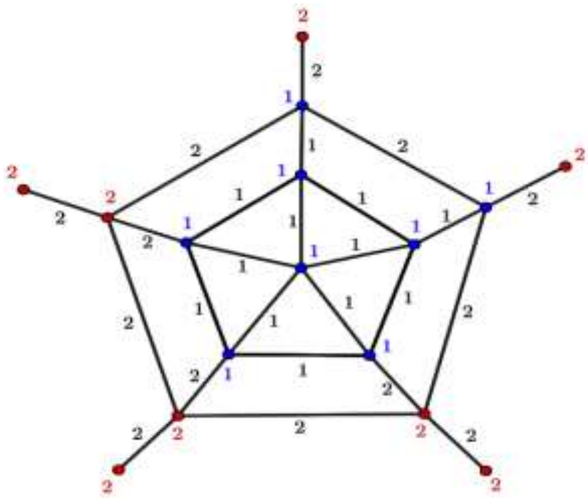
Illustration 2.8. EHMC labeling of Wb_4 and Wb_5 is shown in following figure - 8.



$\boldsymbol{Wb_4}$



$\boldsymbol{Wb_5}$
Figure – 8

Theorem 2.9. The Complete graph K_n is not EHMC for $n \geq 4$.
Proof. Let $G = (V, E) = K_n$ be the Complete graph. Note that $|V| = n$ and $|E| = \frac{n(n-1)}{2}$. Let $V = \{v_1, v_2, \ldots, v_n\}$ be the vertex set and $E = \left\{e_1, e_2, \ldots, e_{\frac{n(n-1)}{2}}\right\}$ be the edge set of K_n.
If possible, let there be a EHMC labeling $f: E(K_n) \to \{1,2\}$ for graph K_n.
If we assign consecutive labeling 2 on $\left\lceil \frac{n(n-1)}{4} \right\rceil$ edges of K_n, then we have $v_f(1) = n - \left\lfloor \frac{n}{4} \right\rfloor$ and $v_f(2) = \left\lfloor \frac{n}{4} \right\rfloor$. Therefore $\left|v_f(1) - v_f(2)\right| = n - 2\left\lfloor \frac{n}{4} \right\rfloor > 2 > 1$.
Without assuming consecutive labeling 2 on $\left\lceil \frac{n(n-1)}{4} \right\rceil$ edges of W_n, we get $\left|v_f(1) - v_f(2)\right| > 2$. Thus K_n does not satisfies EHMC labeling.
Hence, The Complete graph K_n is not EHMC for $n \geq 4$.

Theorem 2.10. The Crown graph $C_n \odot K_1$ is EHMC for any n.
Proof. Let $G = (V, E) = C_n \odot K_1$ be the Crown graph. Note that $|V| = |E| = 2n$. Let $V = \{x, x_1, x_2, \ldots, x_n, x_{n+1}, x_{n+2}, \ldots, x_{2n}\}$ be a vertex set and $E = \{e_1, e_2, \ldots, e_{2n}\}$ be a edge set of $C_n \odot K_1$ shown in the following figure - 9.

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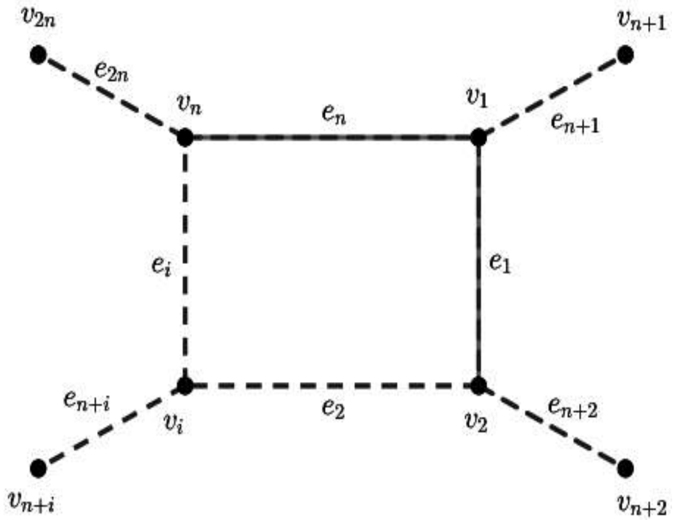

Figure – 9: $\boldsymbol{C_n \odot K_1}$

Define a labeling function $f: E(C_n \odot K_1) \to \{1,2\}$ as follows,
$f(e_i) = 1$ if $1 \leq i \leq n$ and
$f(e_i) = 2$ if $n + 1 \leq i \leq 2n$
Then $v_f(1) = v_f(2) = n$. So, we have $|v_f(1) - v_f(2)| = 0$.
Hence, Crown graph $C_n \odot K_1$ is EHMC for any n.

Illustration 2.11. EHMC labeling of $C_4 \odot K_1$ is shown in following figure - 10.

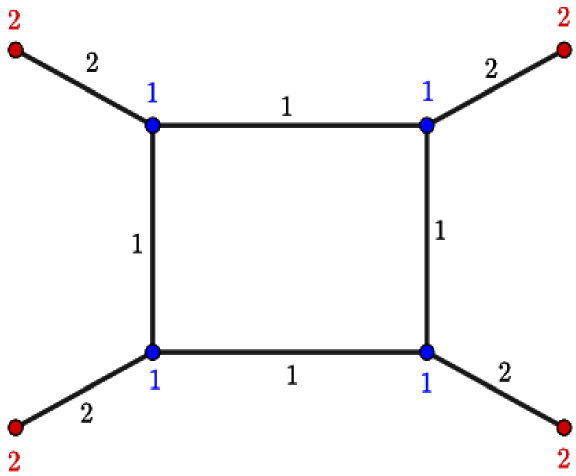

Figure – 10: $\boldsymbol{C_4 \odot K_1}$

Theorem 2.12. The Armed Crown graph ACr_n is EHMC for any n.
Proof. Let $G = (V, E) = ACr_n$ be the armed crown graph. Note that $|V| = |E| = 3n$. Let $V = \{v_1, v_2, \ldots, v_{3n}\}$ be a vertex set and $E = \{e_1, e_2, \ldots, e_{3n}\}$ be an edge set of ACr_n shown in the following figure - 11.

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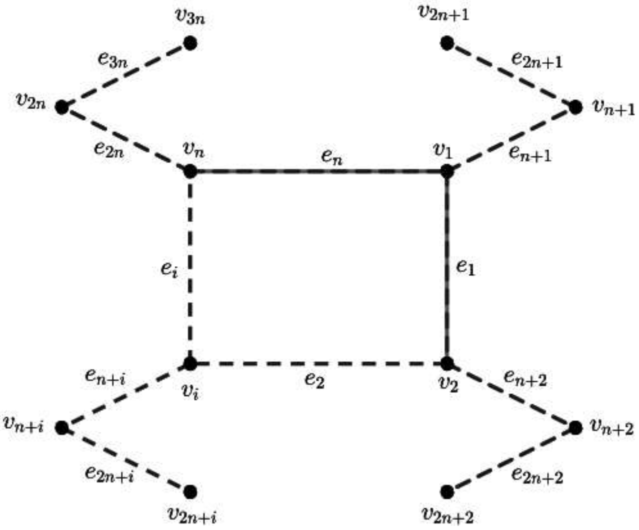
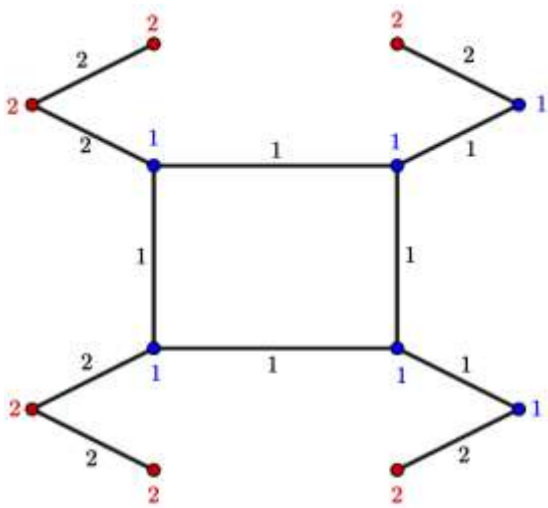


Figure – 11: $\boldsymbol{ACr_n}$

Case 1: n is even
Define a labeling function $f: E(ACr_n) \to \{1,2\}$ as follows,
$f(e_i) = 1$ if $1 \le i \le \frac{3n}{2}$ and
$f(e_i) = 2$ if $\frac{3n+2}{2} \le i \le 3n$
Then $v_f(1) = \frac{3n}{2} = v_f(2)$. So, we have $|v_f(1) - v_f(2)| = 0$.

Case 2: n is odd
Define a labeling function $f: E(ACr_n) \to \{1,2\}$ as follows,
$f(e_i) = 1$ if $1 \le \ i \ \le \left\lfloor \frac{3n}{2} \right\rfloor$ and
$f(e_i) = 2$ if $\left\lfloor \frac{3n}{2} \right\rfloor + 1 \le i \le 5n$
Then $v_f(1) = \frac{3n-1}{2}, v_f(2) = \frac{3n+1}{2}$. So, we have $|v_f(1) - v_f(2)| = 1$.
Hence, Armed Crown graph ACr_n is EHMC.

Illustration 2.13. EHMC labeling of ACr_4 and ACr_5 is shown in following figure - 12.



$\boldsymbol{ACr_4}$

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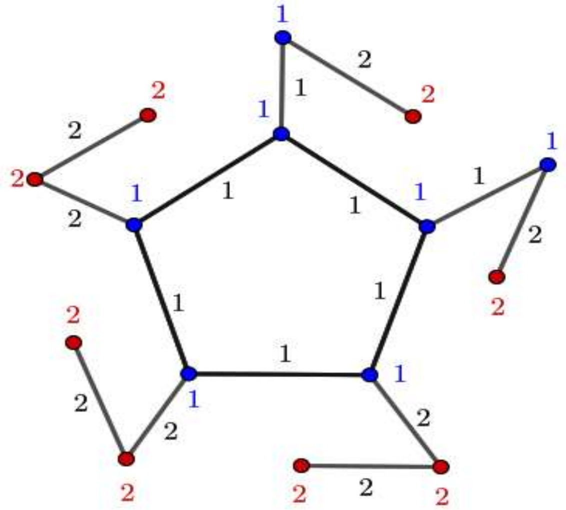

$\boldsymbol{ACr_5}$
Figure – 12

Theorem 2.14. The Flower graph Fl_n is EHMC for any n.
Proof. Let $G = (V, E) = Fl_n$ be the Flower graph. Note that $|V| = 2n + 1$ and $|E| = 4n$. Let $V = \{v, v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}, \ldots, x_{2n}\}$ be a vertex set of G and $E = \{e_1, e_2, \ldots, e_{4n}\}$ be a edge set of G shown in the following figure - 13.

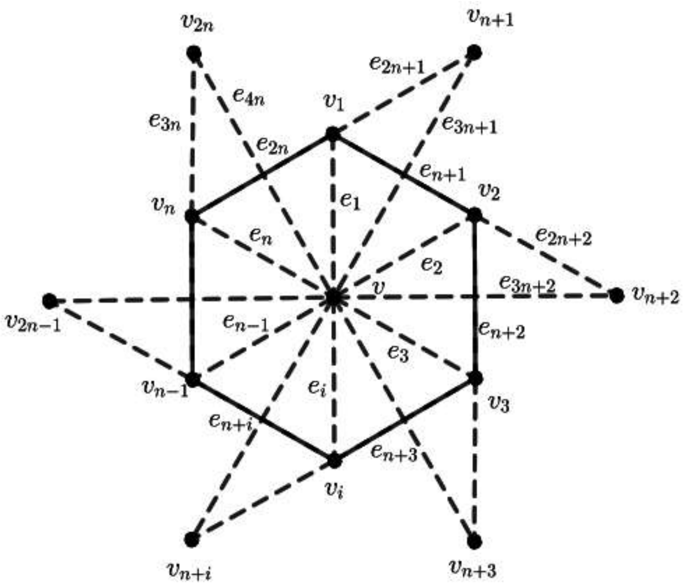

Figure – 13: $\boldsymbol{Fl_n}$

Define a labeling function $f: E(Fl_n) \to \{1,2\}$ as follows,
$f(e_i) = 1$ if $1 \leq i \leq 2n$ and
$f(e_i) = 2$ if $2n + 1 \leq i \leq 4n$
Then $v_f(1) = n + 1, v_f(2) = n$. So, we have $|v_f(1) - v_f(2)| = 1$.
Hence, The Flower graph Fl_n is EHMC for any n.

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Illustration 2.15. EHMC labeling of Fl_6 is shown in following figure - 14.

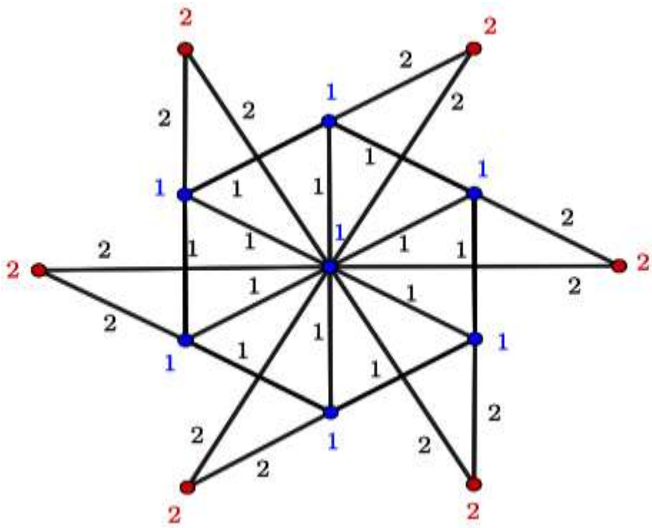

Figure – 14: Fl_6

Theorem 2.16. The Gear graph G_n is EHMC for odd n.
Proof. Let $G = (V, E) = G_n$ be the Gear graph. Note that $|V| = 2n + 1$ and $|E| = 3n$. Let $V = \{v, v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{2n}\}$ be a vertex set and $E = \{e_1, e_2, \dots, e_{3n}\}$ be an edge set of G_n as shown in the following figure - 15.

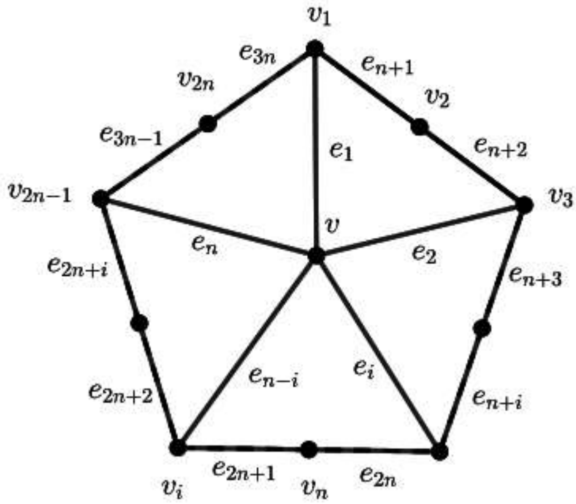

Figure – 15: G_n

Define a labeling function $f: E(G_n) \to \{1,2\}$ as follows,
$f(e_i) = 1$ if $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$,
$f(e_i) = 1$ if $n + 1 \le i \le 2n - 1$
$f(e_i) = 2$ if $\left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n$ and
$f(e_i) = 2$ if $2n \le i \le 3n$
Then $v_f(1) = n + 1, v_f(2) = n$. So, we have $|v_f(1) - v_f(2)| = 1$.
Hence, The Gear graph G_n is EHMC for odd n.

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Illustration 2.17. EHMC labeling of G_5 is shown in following figure - 16.

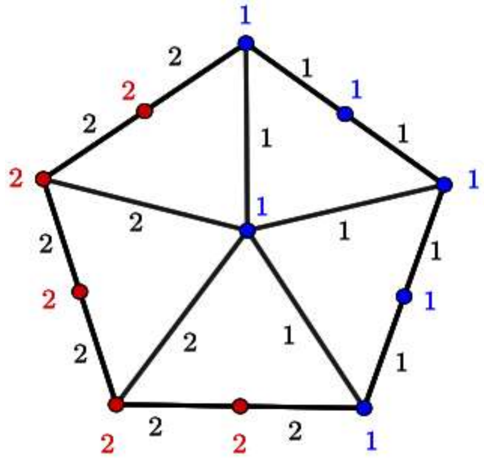

Figure – 16: G_5

Theorem 2.18. The Gear graph G_n is not EHMC for even n.
Proof. Let $G = (V, E) = G_n$ be the Gear graph. Note that $|V| = 2n + 1$ and $|E| = 3n$. Let $V = \{v, v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}, \ldots, v_{2n}\}$ be a vertex set and $E = \{e_1, e_2, \ldots, e_{3n}\}$ be an edge set of G_n.
If possible, let there be a EHMC labeling $f : EG\_n) \rightarrow \{1,2\}$ for graph G_n.
If we assign consecutive labeling 2 on $\frac{3n}{2}$ edges of G_n, then we have $v_f(1) = n + 2$ and $v_f(2) = n - 1$. Therefore $|v_f(1) - v_f(2)| = 3 > 1$.
Without assuming consecutive labeling 2 on $\frac{3n}{2}$ edges of G_n, we get $|v_f(1) - v_f(2)| > 3$. Thus G_n does not satisfies EHMC labeling.
Hence, The Gear graph G_n is not EHMC for even n.

Theorem 2.19. The Shell graph S_n is EHMC for odd n.
Proof. Let $G = (V, E) = S_n$ be the Shell graph. Note that $|V| = n$ and $|E| = 2n - 3$. Let $V = \{x_1, x_2, \ldots, x_n\}$ be a vertex set and $E = \{e_1, e_2, \ldots, e_{2n-3}\}$ be an edge set of S_n shown in the following figure - 17.

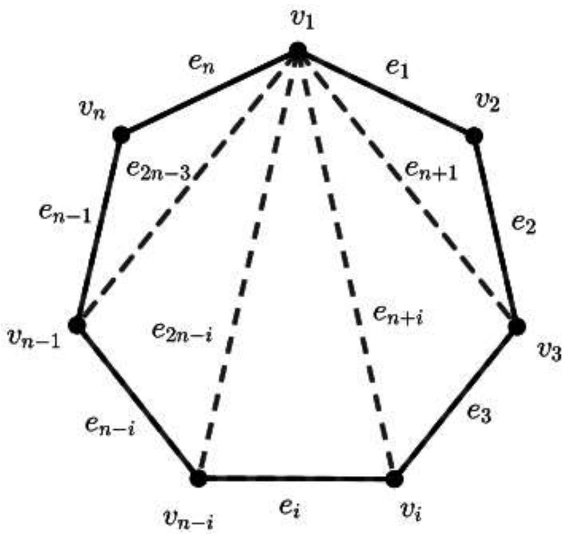

Figure – 17: S_n

Define a labeling function $f: E(S_n) \rightarrow \{1,2\}$ as follows,
$f(e_i) = 1$ if $1 \leq i \leq \frac{n-1}{2}$,
$f(e_i) = 1$ if $n + 1 \leq i \leq \frac{3n-3}{2}$
$f(e_i) = 2$ if $\frac{n+1}{2} \leq i \leq n$ and

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$f(e_i) = 2$ if $\frac{3n-1}{2} \leq i \leq 2n-3$
Then $v_f(1) = \frac{n+1}{2}, v_f(2) = \frac{n-1}{2}$. So, we have $|v_f(1) - v_f(2)| = 1$.
Hence, The Shell graph S_n is EHMC for odd n.

Illustration 2.20. EHMC labeling of S_7 is shown in following figure - 18.

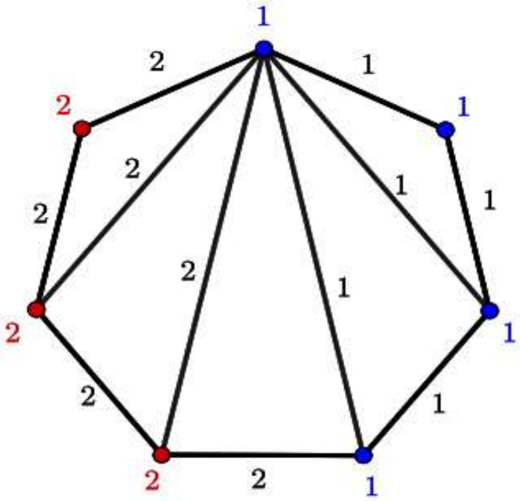

Figure – 18: S_7

Theorem 2.21. The Shell graph S_n is not EHMC for even n.
Proof. Let $G = (V, E) = S_n$ be the Shell graph. Note that $|V| = n$ and $|E| = 2n - 3$. Let $V = \{x_1, x_2, \ldots, x_n\}$ be a vertex set and $E = \{e_1, e_2, \ldots, e_{2n-3}\}$ be an edge set of S_n.
If possible, let there be a EHMC labeling $f: E(S_n) \to \{1,2\}$ for graph S_n.
If we assign consecutive labeling 2 on $\left\lceil\frac{2n-3}{2}\right\rceil$ edges of S_n, then we have $v_f(1) = \frac{n+2}{2}$ and $v_f(2) = \frac{n-2}{2}$. Therefore $|v_f(1) - v_f(2)| = 2 > 1$.
Without assuming consecutive labeling 2 on $\left\lceil\frac{2n-3}{2}\right\rceil$ edges of S_n, we get $|v_f(1) - v_f(2)| > 2$. Thus S_n does not satisfies EHMC labeling.
Hence, The Shell graph S_n is not EHMC for even n.

Conclusion
In this article, we have discussed Cycle C_n for odd n, Helm H_n, Closed helm CH_n, Web Wb_n, Crown $C_n \odot K_1$, Armed Crown ACr_n, Flower Fl_n, Gear G_n for odd n and Shell S_n for odd n are Edge Harmonic Mean Cordial Graphs, respectively. Also, we investigate Cycle C_n for even n, Wheel W_n, Complete graph K_n, Gear G_n for even n and Shell S_n for even n are not Edge Harmonic Mean Cordial Graphs, respectively.

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