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Mathematical Study Of Waiting Time Of Service In Four Server Hierarchical Structured Feedback Queuing System With Revisit At Most Once To Any Of The Servers

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Abstract: The present paper deals with the study of mathematical and graphical study of waiting time of service of a customer. The queuing system has four servers for the service of customers in hierarchical order. A customer, after getting service from a server may leave the system or may go for further service to the higher order server depending upon the need of service but cannot go for service more than two times.
The arrival and service pattern are assumed to follow the Poisson process. The waiting time of customer for the service has been calculated from the mean queue length obtained by using generating function technique.

Keywords: Feedback, Queuing System, Poisson Process, Four Server, Waiting Time.

1. Introduction In the Queueing theory we deal with the study of optimizing the waiting time of service. Generally, queues are formed when the supply of service does not balance with the demand of service. We face queueing problem everywhere in our day-to-day life such as in the banks, post offices, hospitals, airports, highway tolls etc. Many authors did a lot of work on queueing theory.The waiting-time distribution for a finite-space, single-server queueing system, in which customers arrive singly following a Poisson process and the server operates under (a, b)-bulk service rule have analysed. The waiting-time distribution (in the queue) of a random customer is derived using the functional relation between the probability generating function (pgf) for the queue-length distribution and the Laplace–Stieltjes transform (LST) of the queueing-time distribution for a random customer(1). But there are possiblities of queueuing systems having more than one servers. A queueing system consisting of two multi-server subsystems is studied which is designed for the service of clients arriving at a system according to a Markovian arrival process (MAP). Arriving clients receive information about the number of clients present in both subsystems and use this information to make a randomized decision to balk (depart without receiving service) or join the system. If some server in that subsystem is idle during this epoch, the client immediately leaves the buffer where it has been staying and starts a service in the alternative subsystem(2). The Markovian model's quality control policy was studied using an iterative method for the nth customer in the queueing system. A performance measure can be derived for the expected number of units in the system, as well as in the queue and the average number of occupied services and the expected waiting time in the system, as well as in the queue(3). To assess the service efficiency of Ahamadu Bello University Teaching Hospital in Zaria, Kaduna State, an analysis was performed using multiple server queuing models. Primary data was collected through observation and questionnaire methods at the hospital over a two-week period to determine the queuing model that minimizes patients waiting time. Regression analysis showd that all the dimensions of service quality have significant positive relationship with the patient's satisfaction(4). In a queueing system, the Flexible General Bulk Service rule is a threshold that can decrease and regulate the waiting time of customers in any type of bulk service. Future research should not emphasize limited admissibility, service interruptions, or setup time concepts(5). A simple service resetting mechanism can reverse the deleterious effects of large fluctuations in service times, thus turning a marked drawback into a favorable advantage. This happens when stochastic fluctuations are intrinsic to the server, and we show that service resetting can then dramatically cut down average queue lengths and waiting times. These results are illustrated on the M/G/1 queue in which service times are general and arrivals are assumed to be Markovian(6). The waiting time in a queueing system with five parallel servers linked centrally with a common sixth server in the series has been derived(7). A queuing model has been developed for a system having three servers wherein a customer may revisit any of the servers. The visit of the customer is limited to maximum twice. A customer may require the services of one or all the servers. The first server is centrally linked with the other two parallel servers. The customer can enter the system at first server from outside. After getting the service from any of the servers, the customer may revisit to any other server or may leave the system at any stage depending upon his/her satisfaction. The waiting time of the customer is derived from the mean queue length of the system(8). The waiting time of a customer for service in the queuing system with three service channels in hierarchy is analyzed wherein a customer after getting the service from first server may leave the system or move to the second server according as he/she is satisfied with service. However, the customer after getting the service from second or third server may move to any server or may leave the system. Revisit Customers are allowed to request service from any of the servers only once(9). Mean queue length of a four-server hierarchical structured queuing system with feedback and revisit of customer at most once to any of the servers increases and decreases with respect to different parameters(10). The above mentioned researchers have worked on waiting time of customers considering different aspects but none of them considered the fact

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of probability of going of customer from a server of the system having four servers in hierarchy. They also did not assume the probability and frequency of revisit of customer according to her/his need and satisfaction. In the present study, we have derived the waiting time of the customer from the mean queue length of the system by using the Little's formula. The arrival and departing customers behaviour follows the Poisson and exponential distribution respective. Also, we have considered the probability and frequency of revisit of a customer. She/ he may revisit the server at most once with different probability each time. Results of variation in waiting time have been found with respect to different arrival and service rates and probabilities by taking different values of other parameters.

2. Notations

λ: Mean Arrival rate at 1st server (S_1)
μ_1: service rate of 1st server (S_1)
μ_2: service rate of 2nd server.
μ_3: service rate of 3rd server.
μ_4: service rate of 4th server
a_1: the probability of customer leaving 1st server 1st time.
a_2: the probability of customer leaving 1st server 2nd time.
b_1: the probability of customer leaving 2nd server 1st time.
b_2: the probability of customer leaving 2nd server 2nd time.
c_1: the probability of customer leaving 3rd server 1st time.
c_2: the probability of customer leaving 3rd server 2nd time.
d_1: the probability of customer leaving 4th server 1st time.
d_2: the probability of customer leaving 4th server 2nd time.
p_{12}: the probability of customer going from 1st to 2nd server 1st time.
p_1: the probability of exit of customer from 1st server 1st time.
p'_{12} : the probability of customer going from 1st to 2nd server 2nd time.
p'_1 : the probability of exit of customer from 1st server 2nd time.
p_2: the probability of exit of customer from 2nd server 1st time.
p_{23}: the probability of customer going from 2nd to 3rd server 1st time.
p_{21}: the probability of customer going from 2nd to 1st server 1st time.
p'_2 : the probability of exit of customer from 2nd server 2nd time.
p'_{23} : the probability of customer going from 2nd to 3rd server 2nd time.
p'_{21}: the probability of customer going from 2nd to 1st server 2nd time.
p_3: the probability of exit of customer from 3rd server 1st time.
p_{31}: the probability of customer going from 3rd to 1st server 1st time.
p_{32}: the probability of customer going from 3rd to 2nd server 1st time.
p_{34}: the probability of customer going from 3rd to 4th server 1st time
p'_3 : the probability of exit of customer from 3rd server 2nd time.
p'_{31}: the probability of customer going from 3rd to 1st server 2nd time.
p'_{32}:the probability of customer going from 3rd to 2nd server 2nd time.
p'_{34}: the probability of customer going from 3rd to 4th server 2nd time.
p_4 : the probability of exit of customer from 4th server 1st time.
p'_4: the probability of exit of customer from 4th server 2nd time.
p_{41} : the probability of exit of customer from 4th to 1st server 1st time.
p_{42}: the probability of customer going from 4th to 2nd server 1st time.
p_{43}: the probability of customer going from 4th to 3rd server 1st time.
According to the model hypothesis we have:
$a_1p_1 + a_1p_{12} + a_2p_1' + a_2p'_{12} = 1$
$b_1p_2 + b_1p_{23} + b_2p_{21} + b_2p'_2 + b_2p'_{23} = 1$;
$c_1p_3 + c_1p_{31} + c_1p_{32} + c_1p_{34} + c_2p'_3 + c_2p'_{31} + c_2p'_{32} + c_2p'_{34} = 1$;
$d_1p_4 + d_1p_{41} + d_1p_{42} + d_1p_{43} + d_2p'_4 = 1$.

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3. Methodology

The mean queue length obtained in ref. (10) is given by:

$$Lq = -\frac{\lambda}{A}\left[\begin{array}{l} [1-(c_1p_{34}+c_2p_{34}')d_1p_{43}-(b_1p_{23}+b_2p_{23}')(c_1p_{32})-(b_1p_{23}'+b_2p_{23}')(c_1p_{34}+c_2p_{34}')d_1p_{42}] \\ +(a_1p_{12}+a_2p_{12}')[1-(c_1p_{34}+c_2p_{34}')d_1p_{43}]+\dfrac{(a_1p_{12}+a_2p_{12}')(b_1p_{23}+b_2p_{23}')}{[\mu_3-d_1p_{43}\mu_4-(b_1p_{23}+b_2p_{23}')\mu_2]} \\ +\dfrac{(a_1p_{12}+a_2p_{12}')(b_1p_{23}+b_2p_{23}')(c_1p_{34}+c_2p_{34}')}{[\mu_4-\mu_3(c_1p_{34}+c_2p_{34}')]} \end{array}\right]$$

Thus the waiting time of customer for the service in the whole queueing system is:

$$W = -\frac{1}{A}\left[\begin{array}{l} [1-(c_1p_{34}+c_2p_{34}')d_1p_{43}-(b_1p_{23}+b_2p_{23}')(c_1p_{32})-(b_1p_{23}'+b_2p_{23}')(c_1p_{34}+c_2p_{34}')d_1p_{42}] \\ +(a_1p_{12}+a_2p_{12}')[1-(c_1p_{34}+c_2p_{34}')d_1p_{43}]+\dfrac{(a_1p_{12}+a_2p_{12}')(b_1p_{23}+b_2p_{23}')}{[\mu_3-d_1p_{43}\mu_4-(b_1p_{23}+b_2p_{23}')\mu_2]} \\ +\dfrac{(a_1p_{12}+a_2p_{12}')(b_1p_{23}+b_2p_{23}')(c_1p_{34}+c_2p_{34}')}{[\mu_4-\mu_3(c_1p_{34}+c_2p_{34}')]} \end{array}\right]$$

where $A = [(b_1\, p_{21})\, (a_1\, p_{12}' + a_2p_{12}') - 1]\, (c_1\, p_{34} + c_2\, p_{34})\, (d_1\, p_{43} + 1) + (b_1\, p_{23} + b_2\, p_{23}')$
$[c_1\, p_{31} - (c_1\, p_{34} + c_2\, p_{34}')\, d_1p_{41}\, (a_1\, p_{12} + a_2\, p_{12}') + (c_1p_{34} + c_2\, p_{34}')\, d_1\, p_{42}]$

4. Results and Discussion

4.1. Behaviour of waiting time (W) of customer in the system with respect to p_2 (the probability of leaving the system first time from second server) for different values of p_3 (probability of leaving the system from third server first time) is depicted in Table 1 keeping the values of other parameters as fixed.

Table 1. W with respect to p_2 for different values of p_3

$\lambda=2$, $\mu_1 = 10$, $\mu_2 = 7$, $\mu_3= 9$, $\mu_4=8$, $a_1=0.3$, $a_2=0.7$, $b_1=0.1$, $b_2=0.9$, $c_1=0.8$, $c_2=0.2$, $d_1=0.6$, $d_2=0.4$, $p_1=0.7$, $p_{12}=0.3$, $p_1'=0.4$, $p_{12}'=0.6$, $p_3=0.3$, $p_{31}=0.2$, $p_{32}=0.1$, $p_{34}=0.4$, $p_3'=7$, $p'_{34}=0.3$, $p_4=0.5$, $p_{41}=0.1$, $p_{42}=0.2$, $p_{43}=0.2$, $p'_4=1$			
	$p_3=0.3$	$p_3=0.4$	$p_3=0.45$
p_2	W	W	W
0.3	3.82978093	11.21512736	22.18906615
0.325	3.820670698	11.23123064	22.35192764
0.35	3.811606405	11.24739183	22.51724962
0.375	3.802587754	11.26361126	22.68508799
0.4	3.79361445	11.27988928	22.85550036
0.425	3.784686201	11.29622623	23.02854612
0.45	3.775802719	11.31262244	23.20428648
0.475	3.766963718	11.32907828	23.38278456
0.5	3.758168914	11.34559408	23.56410549
0.525	3.74941803	11.3621702	23.74831643
0.55	3.740710786	11.37880699	23.9354867

Following can be interpreted from **Table 1**:
(i) Waiting time decreases with the increase in the probability p_2 for the probability $p_3 \leq 0.3$ and increases for $p_3>0.3$.
(ii) Waiting time increases with the increase in the probability p_3.

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4.2. Behaviour of waiting time (W) of customer in the system with respect to p_1 (the probability of leaving the system from first server first time) for different values of p'_1 (probability of leaving the system from first server second time) is depicted in Table 2 keeping the values of other parameters as fixed.

Table 2. W with respect to p_1 for different values of p'_1

$\lambda=2$, $\mu_1 = 10$, $\mu_2 = 7$, $\mu_3 = 9$, $\mu_4=8$, $a_1=0.3$, $a_2=0.7$, $b_1=0.1$, $b_2=0.9$, $c_1=0.8$, $c_2=0.2$, $d_1=0.6$, $d_2=0.4$, $p_2= 0.3$, $p_{21}=0.2$, $p_{23}=0.5$, $p'_2=0.8$, $p'_{23}=0.2$, $p_3=0.3$, $p_{31}=0.2$, $p_{32}=0.1$, $p_{34}=0.4$, $p_3'=7$, $p'_{34}=0.3$, $p_4=0.5$, $p_{41}=0.1$, $p_{42}=0.2$, $p_{43}=0.2$, $p'_4=1$			
	$p'_1=0.4$	$p'_1=0.6$	$p'_1=0.8$
p_1	W	W	W
0.1	4.280994091	3.90563365	3.531853017
0.15	4.240352722	3.865126594	3.491479505
0.2	4.199721901	3.82463003	3.451116428
0.25	4.159101626	3.784143954	3.410763782
0.3	4.118491892	3.743668362	3.370421564
0.35	4.077892694	3.703203249	3.330089769
0.4	4.037304029	3.662748613	3.289768394
0.45	3.996725893	3.622304448	3.249457434
0.5	3.956158281	3.58187075	3.209156884
0.55	3.915601189	3.541447516	3.168866743
0.6	3.875054614	3.501034742	3.128587004

Following can be interpreted from **Table 2**:
Waiting time decreases with the increase in the probability p_1 as well as for p'_1.

4.3. Behaviour of waiting time (W) of customer in the system with respect to p_1 (the probability of leaving the system from first server first time) for different values of μ_2 (service rate of second server) is depicted in Table 3 keeping the values of other parameters as fixed.

Table 3. W with respect to p_1 for different values of μ_2

$\lambda=2$, $\mu_1 = 10$, $\mu_2 = 7$, $\mu_3 = 9$, $\mu_4=8$, $a_1=0.3$, $a_2=0.7$, $b_1=0.1$, $b_2=0.9$, $c_1=0.8$, $c_2=0.2$, $d_1=0.6$, $d_2=0.4$, $p_1'=0.4$, $p_{12}'=0.6$, $p_2= 0.3$, $p_{21}=0.2$, $p_{23}=0.5$, $p'_2=0.8$, $p'_{23}=0.2$, $p_3=0.3$, $p_{31}=0.2$, $p_{32}=0.1$, $p_{34}=0.4$, $p_3'=7$, $p'_{34}=0.3$, $p_4=0.5$, $p_{41}=0.1$, $p_{42}=0.2$, $p_{43}=0.2$, $p'_4=1$			
	$\mu_2=7$	$\mu_2=14$	$\mu_2=21$
p_1	W	W	W
0.2	4.199721901	4.220599547	4.262419878
0.25	4.159101626	4.179502132	4.220366697
0.3	4.118491892	4.138415382	4.17832443
0.35	4.077892694	4.097339293	4.136293071
0.4	4.037304029	4.056273859	4.094272616
0.45	3.996725893	4.015219078	4.05226306
0.5	3.956158281	3.974174945	4.010264401
0.55	3.915601189	3.933141456	3.968276633
0.6	3.875054614	3.892118607	3.926299753
0.65	3.834518551	3.851106394	3.884333755
0.7	3.793992996	3.810104812	3.842378636

Following can be interpreted from **Table 3**:
(i) Waiting time decreases with the increase in the probability p_1 .
(ii) Waiting time increases with the increase in the service rate of second server i. e. μ_2.

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4.4. Behaviour of waiting time (W) of customer in the system with respect to μ_1 (the service rate of first server) for different values of μ_4 (service rate of fourth server) is depicted in Table 4 keeping the values of other parameters as fixed.

Table 4. W with respect to μ_1 for different values of μ_4

$\lambda=2$, $\mu_2 = 7$, $\mu_3= 9$, $a_1=0.3, a_2=0.7$, $b_1=0.1$, $b_2=0.9$, $c_1=0.8$, $c_2=0.2$, $d_1=0.6$, $d_2=0.4, p_1=0.7$, $p_{12}=0.3$, $p_1'=0.4, p_{12}'=0.6$, $p_2=0.3$, $p_{21}=0.2$, $p_{23}=0.5, p'_2=0.8$, $p'_{23}=0.2$, $p_3=0.3$, $p_{31}=0.2$, $p_{32}=0.1$, $p_{34}=0.4$, $p_3'=7, p'_{34}=0.3, p_4=0.5$, $p_{41}=0.1$, $p_{42}=0.2$, $p_{43}=0.2$, $p'_4=1$			
	$\mu_4=8$	$\mu_4=9$	$\mu_4=10$
μ_1	W	W	W
1	3.793992996	3.792830525	3.792335773
2	3.793992996	3.792830525	3.792335773
3	3.793992996	3.792830525	3.792335773
4	3.793992996	3.792830525	3.792335773
5	3.793992996	3.792830525	3.792335773
6	3.793992996	3.792830525	3.792335773
7	3.793992996	3.792830525	3.792335773
8	3.793992996	3.792830525	3.792335773
9	3.793992996	3.792830525	3.792335773
10	3.793992996	3.792830525	3.792335773
11	3.793992996	3.792830525	3.792335773

Following can be interpreted from **Table 4**:
(i) Waiting time remains constant with the increase in service rate of first servre i.e. μ_1.
(ii) Waiting time decreases with the increase in service rate of the fourth server i.e. μ_4.

4.5. Behaviour of waiting time (W) of customer in the system with respect to μ_3 (the service rate of third server) for different values of μ_4 (service rate of fourth server) is depicted in Table 5 keeping the values of other parameters as fixed.

Table 5. W with respect to μ_3 for different values of μ_4

$\lambda=2$, $\mu_1=9, \mu_2 = 7$, $a_1=0.3, a_2=0.7$, $b_1=0.1$, $b_2=0.9$, $c_1=0.8$, $c_2=0.2$, $d_1=0.6$, $d_2=0.4, p_1=0.7$, $p_{12}=0.3$, $p_1'=0.4, p_{12}'=0.6$, $p_2=0.3$, $p_{21}=0.2$, $p_{23}=0.5, p'_2=0.8$, $p'_{23}=0.2$, $p_3=0.3$, $p_{31}=0.2$, $p_{32}=0.1$, $p_{34}=0.4$, $p_3'=7, p'_{34}=0.3, p_4=0.5$, $p_{41}=0.1$, $p_{42}=0.2$, $p_{43}=0.2$, $p'_4=1$			
	$\mu_4=8$	$\mu_4=9$	$\mu_4=10$
μ_3	W	W	W
4	3.959245393	3.978016645	4.001053484
5	3.870501003	3.87590416	3.88233696
6	3.833868446	3.83576387	3.838264163
7	3.814118013	3.814497349	3.81541612
8	3.801987014	3.801479827	3.801548997
9	3.793992996	3.792830525	3.792335773
10	3.788551015	3.786802579	3.785862078
11	3.78485814	3.782502688	3.781154812
12	3.782493609	3.779437564	3.777671993
13	3.781258962	3.777326781	3.775093775
14	3.78112022	3.776018789	3.773226668

Following can be interpreted from **Table 5**:
(i) Waiting time decreases with the increase in service rate of third servre i.e. μ_3.
(ii) Waiting time increases with the increase in service rate of the fourth server i.e. μ_4.

4.6. Behaviour of waiting time (W) of customer in the system with respect to μ_4 (the service rate of fourth server) for different values of μ_2 (service rate of second server) is depicted in Table 6 keeping the values of other parameters as fixed.

Table 6. W with respect to μ_4 for different values of μ_2

$\lambda=2$, $\mu_1=9$, $\mu_3=7$, $a_1=0.3$, $a_2=0.7$, $b_1=0.1$, $b_2=0.9$, $c_1=0.8$, $c_2=0.2$, $d_1=0.6$, $d_2=0.4$, $p_1=0.7$, $p_{12}=0.3$, $p_1'=0.4$, $p_{12}'=0.6$, $p_2=0.3$, $p_{21}=0.2$, $p_{23}=0.5$, $p'_2=0.8$, $p'_{23}=0.2$, $p_3=0.3$, $p_{31}=0.2$, $p_{32}=0.1$, $p_{34}=0.4$, $p_3'=7$, $p'_{34}=0.3$, $p_4=0.5$, $p_{41}=0.1$, $p_{42}=0.2$, $p_{43}=0.2$, $p'_4=1$			
	$\mu_2=8$	$\mu_2=9$	$\mu_2=10$
μ_4	W	W	W
1	3.797488641	3.797044506	3.797076662
2	3.799689003	3.799340653	3.799474987
3	3.802078555	3.801838755	3.802089208
4	3.804682793	3.804566682	3.804949852
5	3.807532011	3.807557679	3.808093484
6	3.810662482	3.81085173	3.811564281
7	3.814118013	3.814497349	3.81541612
8	3.817952007	3.818553994	3.819715396
9	3.822230242	3.82309533	3.824544876
10	3.827034678	3.828213741	3.830009089
11	3.832468754	3.834026685	3.836241967

Following can be interpreted from **Table 6**:
(i) Waiting time increases with the increase in service rate of fourth servre i.e. μ_4.
(ii) Waiting time decreases for $\mu_2 \leq 9$ and increases for $\mu_2 > 9$.

5. Conclusion

Behaviour of waiting time (W) of customer in the system with respect to different rates and probabilities depicted in the tables from 1 to 6 keeping the values of other parameters as fixed is according to the following table:

Result From	Result With Respect to	Result of Waiting Time
Table 1	Increase in probability p_2 for $p_3>0.3$	Increase
	Increase in probability p_2 for the probability $p_3\leq0.3$	Decrease
	increase in the probability p_3	Increase
Table 2	increase in the probability p_1	Increase
	increase in the probability p'_1	Increase
Table 3	increase in the probability p_1	Decreases
	increase in the service rate of second server i. e. μ_2	Increases
Table 4	increase in service rate of first servre i.e. μ_1.	Constant
	increase in service rate of the fourth server i.e. μ_4.	Decreases
Table 5	increase in service rate of third servre i.e. μ_3	Decreases
	increase in service rate of the fourth server i.e. μ_4.	Increases
Table 6	increase in service rate of fourth servre i.e. μ_4	Increases
	Upto $\mu_2\leq 9$	Decreases
	for $\mu_2> 9$	Increases

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