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Numerical And Graphical Study Of The Average Queue Length For A Hierarchical Feedback Queuing System With Four Servers And The Limited Number Of Revisits To Any One Server

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Abstract
In this paper, a feedback queueing model that takes into account four servers, connected in hierarchical order. Customers enter the system only through the first server and may proceed to the second, third, or fourth server after that from lower order to higher order server. The customer may return to the previously visited server up to limited number of times if the service does not meet their needs. With each revisit, the chances of returning to the servers are considered to be distinct. The variations in system's average mean queue lengths have been determined using numericals and graphics.

Key Words: *Feedback Queuing System, Four Types of Servers, Hierarchical Order, Limited Revisits to Servers.*

1. Introduction: Many authors worked on queuing theory having feedback facility. Y.E. Lee, B.D. Choi and Y.C. Kim (1998) worked on retrial queue with geometric loss and feedback. They found the mean queue length of the system using the joint generating function. Ankita Roy Chowdhury and Indra Rani (2021) found the steady state solution of catastrophic feedback queue subject to balking using matrix-geometric approach. A. Nazarov, A. Melikov, E. Pavlova et al. analyse an M/M/N Queuing system with feedback by the method of asymptotic analysis.

Kusum et al. (2012) worked on feedback queues having three service channels wherein a customer may go forward/back to any service channel. But there is no restriction on the number of such movements. Also, they considered the same probability on every revisit. Kumar and Taneja (2017), worked on the feedback queuing system comprising of three servers linked in series hierarchically in which a customer firstly join the first server, then either he/she may leave the system after getting the service or may move to the second higher ordered server for further service. From the second server either he/she may go outside the system or back to the first lower ordered server or may go to the third highest ordered server for further service depending upon the need of customer. From the third highest ordered server he/she may go outside the system or to the second server or to the first server. Here it is assumed that the customer may revisit any server at most once and once he/she reached the third server second time, he/she will quit the system. However, they did not discuss about the situation when the number of servers is more than three.

Kamal et al. (2023) worked on hierarchically structured four server feedback queueing system. But they assumed the revisit only once. There may be systems in place where services are provided in a hierarchical manner having four servers with the provision of service more than twice; as a result, Nidhi et al. (2024) dedicated to examining these systems. They assumed the revisits up to limited number of times. Administrative offices, medical facilities, and hierarchical organizations may all encounter this kind of circumstance.

As a result, they discussed a queue system where customers can either proceed to the second higher level server based on their pleasure with the service, or they can exit the system after the first server completes their task. The customer may move to the next higher-level third server for more service after receiving care from the second server. There's also a possibility that he or she will quit the system or come back to the original server with criticism. If the customer is not satisfied, they can leave the system after being satisfied or they can go from this server to any of the lower-level servers. In this instance, we have taken into account the circumstances in which a client is obligated to return up to a certain number of times. Each time you come back, your probability of getting on any given server are considered to be unique. With the use of the differential-difference method, the queue lengths have been established by them. But they did not justify the findings numerically and graphically.

In this study, we have provided a numerical and graphical explanation of the model's results by giving different variables and queueing characteristics arbitrary numerical values in the derived formulae.

2. Notations:

λ: Mean Arrival rate at 1st server (S_1)
μ_1: service rate of 1st server (S_1)
μ_2: service rate of 2nd server (S_2)
μ_3: service rate of 3rd server (S_3)
μ_4: service rate of 4th server (S_4)

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p^i_{12}: the probability of customer going from 1st to 2nd server ith time
p^i_1: the probability of exit of customer from 1st server ith time.
p^i_2: the probability of exit of customer from 2nd server ith time.
p^i_{23}: the probability of customer going from 2nd to 3rd server ith time.
p^i_{21}: the probability of customer going from 2nd to 1st server ith time.
p^i_3: the probability of exit of customer from 3rd server ith time.
p^i_{31}: the probability of customer going from 3rd to 1st server ith time.
p^i_{32}: the probability of customer going from 3rd to 2nd server ith time.
p^i_{34}: the probability of customer going from 3rd to 4th server ith time.
p^i_4: the probability of exit of customer from 4th server ith time.
p^i_{41}: the probability of exit of customer from 4th to 1st server ith time.
p^i_{42}: the probability of customer going from 4th to 2nd server ith time.
p^i_{43}: the probability of customer going from 4th to 3rd server ith time.

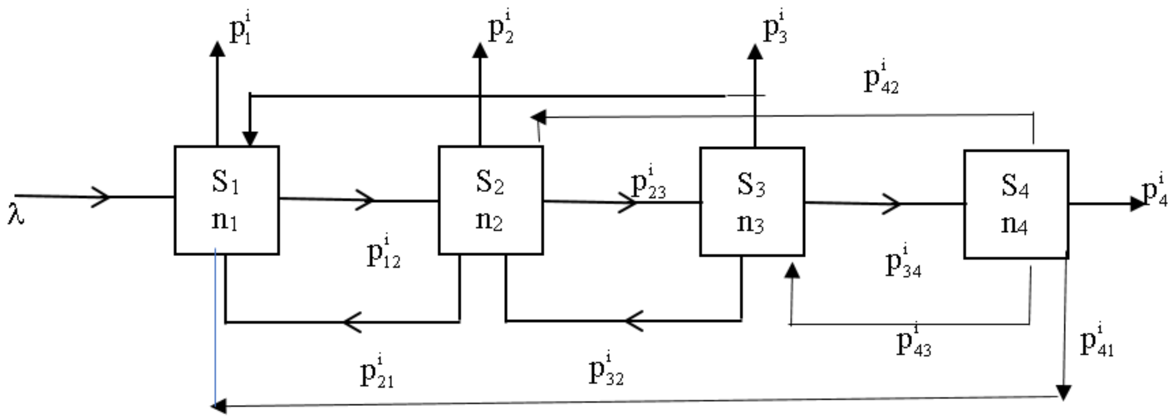
$$A_1=\sum_{i=1}^{n}a^i p_1^i\ ,\ A_{12}=\sum_{i=1}^{n}a^i p_{12}^i$$

$$B_2=\sum_{i=1}^{n}b^i p_2^i\ ,\ B_{21}=\sum_{i=1}^{n-1}b^i p_{21}^i\ ,\ B_{23}=\sum_{i=1}^{n}b^i p_{23}^i$$

$$C_3=\sum_{i=1}^{n}c^i p_3^i\ ,\ C_{34}=\sum_{i=1}^{n}c^i p_{34}^i\ ,\ C_{31}=\sum_{i=1}^{n-1}c^i p_{31}^i\ ,\ C_{32}=\sum_{i=1}^{n-1}c^i p_{32}^i$$

$$D_4=\sum_{i=1}^{n}d^i p_4^i\ ,\ D_{43}=\sum_{i=1}^{n-1}d^i p_{43}^i\ ,\ D_{42}=\sum_{i=1}^{n-1}d^i p_{42}^i\ ,D_{41}=\sum_{i=1}^{n-1}d^i p_{41}^i$$

3. **Formulation of Problem:** The queue network consists of four service channels in hierarchical order i.e. lower level (Server 1) to higher levels (Server 2, server 3 and then server 4) if required. It is assumed that customer arrives at first server from outside the system and then goes to second, third and fourth server. The situation has been shown by the following sate transition diagram:



Movement of the Customers from Various Servers

If the customer gets service from first server ith time, then $p_1^i+p_{12}^i=1$. After getting service from second server ith time, we have $p_2^i+p_{23}^i+p_{21}^i=1$. Customer after getting service from the third server ith time, we have $p_3^i+p_{31}^i+p_{32}^i+p_{34}^i=1$ and after from fourth server, we have $p_4^i+p_{43}^i+p_{42}^i+p_{41}^i=1$.
Hence we can write:

$$\sum_{i=1}^{n}a^i p_1^i+\sum_{i=1}^{n}a^i p_{12}^i=1$$

$$\sum_{i=1}^{n} b^i p_2^i + \sum_{i=1}^{n-1} b^i p_{21}^i + \sum_{i=1}^{n} b^i p_{23}^i = 1$$

$$\sum_{i=1}^{n} c^i p_3^i + \sum_{i=1}^{n} c^i p_{34}^i + \sum_{i=1}^{n-1} c^i p_{31}^i + \sum_{i=1}^{n-1} c^i p_{32}^i = 1$$

$$\sum_{i=1}^{n} d^i p_4^i + \sum_{i=1}^{n-1} d^i p_{43}^i + \sum_{i=1}^{n-1} d^i p_{42}^i + \sum_{i=1}^{n-1} d^i p_{41}^i = 1$$

Thus we have:
$A_1 + A_{12} = 1$
$B_2 + B_{23} + B_{21} = 1$
$C_3 + C_{34} + C_{31} + C_{32} = 1$
$D_4 + D_{43} + D_{42} + D_{41} = 1$

We also define the generating function as $F(X,Y,Z,R) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty} Q_{n_1,n_2,n_3,n_4} X^{n_1} Y^{n_2} Z^{n_3} R^{n_4}$

Where

$$|X| = |Y| = |Z| = |R| = 1 \qquad \ldots(1)$$

Also we define partial generating function as

$$G_{n_2,n_3,n_4}(X) = \sum_{n_1=0}^{\infty} Q_{n_1,n_2,n_3,n_4} X^{n_1} \qquad \ldots(2)$$

$$G_{n_3,n_4}(X,Y) = \sum_{n_2=0}^{\infty} G_{n_2,n_3,n_4}(x).\, Y^{n_2} \qquad \ldots(3)$$

$$G_{n_1,n_3,n_4}(Y) = \sum_{n_2=0}^{\infty} G_{n_1,n_2,n_3,n_4} Y^{n_2} \qquad \ldots(4)$$

$$G_{n_4}(X,Y,Z) = \sum_{n_3=0}^{\infty} G_{n_3,n_4}(X,Y).\, Z^{n_3} \qquad \ldots(5)$$

$$G(X,Y,Z) = \sum_{n_4=0}^{\infty} G_{n_4}(X,Y,Z).\, R^{n_4} \qquad \ldots(6)$$

$$G(X,Y,Z,R) = \frac{\begin{matrix}\mu_1 G_0(Y,Z,R)\left[1-\frac{1}{X}(A_1+A_{12}Y)\right] \\ +\mu_2 G_0(X,Z,R)\left[1-\frac{1}{Y}(B_2+B_{21}X+B_{23}Z)\right]) \\ +\mu_3 G_0(X,Y,R)\left[1-\frac{1}{Z}(C_3+C_{31}X+C_{32}Y+C_{34}R)\right]) \\ +\mu_4 G_0(X,Y,Z)\left[1-\frac{1}{R}(D_4+D_{41}X+D_{42}Y+D_{43}Z)\right])\end{matrix}}{\begin{matrix}\lambda(1-X)+\mu_1\left[1-\frac{1}{X}(A_1+A_{12}Y)\right] \\ +\mu_2\left[1-\frac{1}{Y}(B_2+B_{21}X+B_{23}Z)\right] \\ +\mu_3\left[1-\frac{1}{Z}(C_3+C_{31}X+C_{32}Y+C_{34}R)\right] \\ +\mu_4\left[1-\frac{1}{R}(D_4+D_{41}X+D_{42}Y+D_{43}Z)\right]\end{matrix}} \qquad (7)$$

$$where\ f = \begin{cases}\mu_1 G_0(Y,Z,R)\left[1-\frac{1}{X}(A_1+A_{12}Y)\right] \\ +\mu_2 G_0(X,Z,R)\left[1-\frac{1}{Y}(B_2+B_{21}X+B_{23}Z)\right]) \\ +\mu_3 G_0(X,Y,R)\left[1-\frac{1}{Z}(C_3+C_{31}X+C_{32}Y+C_{34}R)\right]) \\ +\mu_4 G_0(X,Y,Z)\left[1-\frac{1}{R}(D_4+D_{41}X+D_{42}Y+D_{43}Z)\right])\end{cases} \qquad (8)$$

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$$g = \begin{cases} \lambda(1-X) + \mu_1\left[1 - \frac{1}{X}(A_1 + A_{12}Y)\right] \\ +\mu_2\left[1 - \frac{1}{Y}(B_2 + B_{21}X + B_{23}Z)\right] \\ +\mu_3\left[1 - \frac{1}{Z}(C_3 + C_{31}X + C_{32}Y + C_{34}R)\right] \\ +\mu_4\left[1 - \frac{1}{R}(D_4 + D_{41}X + D_{42}Y + D_{43}Z)\right] \end{cases} \tag{9}$$

for convenience let us defined

$$G_0(Y,Z,R) = G_1 \quad , \quad G_0(X,Z,R) = G_2$$
$$G_0(X,Y,R) = G_3 \qquad G_0(X,Y,Z) = G_4 \tag{10}$$

for X = Y = Z = R = 1
Solving the linear equations we have:

$$G_1 = G_0(Y,Z,R) = \frac{\begin{matrix}-(D_{43}C_{34}+D_{42}B_{23}C_{34}+C_{32}B_{23-1})\lambda+ \\ \mu_1B_{21}A_{12}(D_{43}C_{34}-1)+\mu_1 23(+1-D_{43}C_{34}-D_{42}B_{23}C_{34}-C_{32}B_{23}+1) \\ -\mu_1D_{41}A_{12}B_{23}C_{34}-\mu_1C_{31}A_{12}B_{23}\end{matrix}}{\begin{matrix}\mu_1(D_{43}C_{34}+D_{42}B_{23}C_{34}+C_{32}B_{23-1})+ \\ \mu_1B_{21}A_{12}(1-D_{43}C_{34})+\mu_1D_{41}A_{12}B_{23}C_{34} \\ +\mu_1C_{31}A_{12}B_{23}\end{matrix}} \tag{11}$$

$$G_2 = G_0(X,Z,R) = \frac{\begin{matrix}-A_{12}(D_{43}C_{34}-1)\lambda+\mu_2B_{21}A_{12}(D_{43}C_{34}-1) \\ +\mu_2(-D_{43}C_{34}-D_{42}B_{23}C_{34}-C_{32}B_{23}+1) \\ -\mu_2D_{41}A_{12}B_{23}C_{34}-\mu_2C_{31}A_{12}B_{23}\end{matrix}}{\begin{matrix}\mu_2(D_{43}C_{34}+D_{42}B_{23}C_{34}+C_{32}B_{23}-1) \\ +\mu_2B_{21}A_{12}(1-D_{43}C_{34})+\mu_2D_{41}A_{12}B_{23}C_{34} \\ +\mu_2C_{31}A_{12}B_{23}\end{matrix}} \tag{12}$$

$$G_3 = G_0(X,Y,R) = \frac{\begin{matrix}A_{12}B_{23}\lambda+\mu_3(D_{43}C_{34}+D_{42}B_{23}C_{34}+C_{32}B_{23}-1) \\ \mu_3B_{21}A_{12}(1-D_{43}C_{34})+\mu_3D_{41}A_{12}B_{23}C_{34} \\ +\mu_3C_{31}A_{12}B_{23}\end{matrix}}{\begin{matrix}\mu_3(D_{43}C_{34}+D_{42}B_{23}C_{34}+C_{32}B_{23}-1) \\ +\mu_3B_{21}A_{12}(1-D_{43}C_{34})+\mu_3D_{41}A_{12}B_{23}C_{34} \\ +\mu_3C_{31}A_{12}B_{23}\end{matrix}} \tag{13}$$

$$G_4 = G_0(X,Y,Z) = \frac{\begin{matrix}A_{12}B_{23}C_{34}\lambda+\mu_4(D_{43}C_{34}+D_{42}B_{23}C_{34}+C_{32}B_{23}-1) \\ +\mu_4B_{21}A_{12}(1-D_{43}C_{34})+\mu_4D_{41}A_{12}B_{23}C_{34} \\ +\mu_4C_{31}A_{12}B_{23}\end{matrix}}{\begin{matrix}\mu_4(D_{43}C_{34}+D_{42}B_{23}C_{34}+C_{32}B_{23}-1) \\ +\mu_4B_{21}A_{12}(1-D_{43}C_{34})+\mu_4D_{41}A_{12}B_{23}C_{34} \\ +\mu_4C_{31}A_{12}B_{23}\end{matrix}} \tag{14}$$

If Lq be the mean queue length of the whole system then

$$\begin{aligned} Lq = -\mu_1 &\left[\frac{(\mu_1G_1 - \mu_2B_{21}G_2 - \mu_3C_{31}G_3 - \mu_4D_{41}G_4)}{(-\lambda + \mu_1 - \mu_2B_{21} - \mu_3C_{31} - \mu_4D_{41})^2} + \frac{G_1}{(-\lambda + \mu_1 - \mu_2B_{21} - \mu_3C_{31} - \mu_4D_{41})}\right] \\ &- \mu_2\left[\frac{(-\mu_1G_1A_{12} + \mu_2G_2 - \mu_4D_{42}G_4)}{(-A_{12}\mu_1 + \mu_2 - \mu_3C_{32} - \mu_4D_{42})^2} + \frac{G_2}{(-A_{12}\mu_1 + \mu_2 - \mu_3C_{32} - \mu_4D_{42})}\right] \\ &- \mu_3\left[\frac{(-\mu_2B_{23}G_2 + \mu_3G_3)}{(-\mu_2B_{23} + \mu_3 - D_{43}\mu_4)^2} + \frac{G_3}{(-\mu_2B_{23} + \mu_4 - \mu_4D_{43})}\right] \\ &- \mu_4\left[\frac{(-\mu_3C_{34}G_3 + \mu_4G_4)}{(-\mu_3C_{34} + \mu_4)^2} + \frac{G_4}{-\mu_3C_{34} + \mu_4}\right] \end{aligned}$$

… (15)

REDVET - Revista electrónica de Veterinaria - ISSN 1695-7504
Vol 25, No. 2 (2024)
http://www.veterinaria.org
Article Received: Revised: Accepted:

4. Numerical Results and Discussion:

4.1 Behaviour of mean queue length of the system (L_q) with respect to arrival rate (λ) for different values of probability A_1 is depicted in Table 4.1 and in Fig. 4.1 keeping the values of other parameters as fixed.

Table 4.1

μ_1 = 2, μ_2 = 3, μ_3= 4, μ_4= 1, B_2=0.5, B_{21}=0.2, B_{23}=0.3, C_3=0.6, C_{32}=0.15, C_{31}=0.2, C_{34}=0.05, D_4=0.5, D_{41}=0.3, D_{42}=0.15, D_{43}=0.05			
	A_1=0.1	**A_1=0.5**	**A_1=0.8**
λ	**L_q**	**L_q**	**L_q**
50	193.6819	28.75724	4.469178
51	196.9118	29.33559	4.61965
52	200.1417	29.91388	4.770075
53	203.3716	30.49212	4.920456
54	206.6014	31.07032	5.070794
55	209.8311	31.64847	5.221093
56	213.0608	32.22658	5.371354
57	216.2904	32.80464	5.52158
58	219.52	33.38267	5.671772
59	222.7496	33.96067	5.821932
60	225.9791	34.53863	5.972061

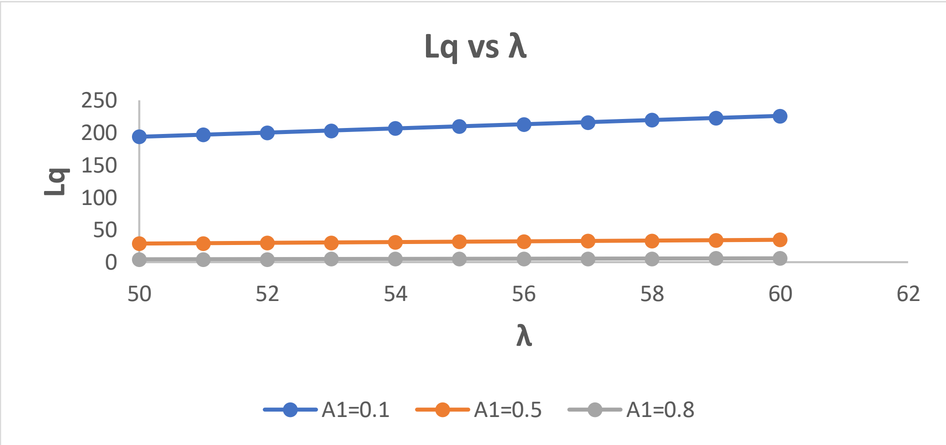

Fig. 4.1

Following can be interpreted from **Table 4.1** and **Fig. 4.1**:
(i) Mean queue length (L_q) get increased with the increase in λ.
(ii) Mean queue length (L_q) decreases with respect to the increase in probability (A_1).

4.2 Behaviour of mean queue length of the system (L_q) with respect to probability A_1 for different values of arrival rate (λ) is depicted in Table 4.2 and in Fig. 4.2 keeping the values of other parameters as fixed.

Table 4.2

μ_1 = 2, μ_2 = 3,μ_3= 4, μ_4= 1, B_2=0.5, B_{21}=0.2, B_{23}=0.3, C_3=0.6, C_{32}=0.15, C_{31}=0.2, C_{34}=0.05, D_4=0.5, D_{41}=0.3, D_{42}=0.15, D_{43}=0.05			
	λ=10	**λ=14**	**λ=16**
A_1	**L_q**	**L_q**	**L_q**
0.1	64.20122	77.24253	83.73823
0.15	44.35484	54.22235	59.13165
0.2	32.04409	39.78114	43.6257
0.25	23.7716	29.98329	33.06565
0.3	17.88556	22.95355	25.46452
0.35	13.51438	17.6946	19.76212
0.4	10.15872	13.63098	15.34495
0.45	7.51371	10.4091	11.83504
0.5	5.383539	7.800484	8.987615
0.55	3.637143	5.651405	6.637587
0.6	2.183733	3.85483	4.669811

REDVET - Revista electrónica de Veterinaria - ISSN 1695-7504
Vol 25, No. 2 (2024)
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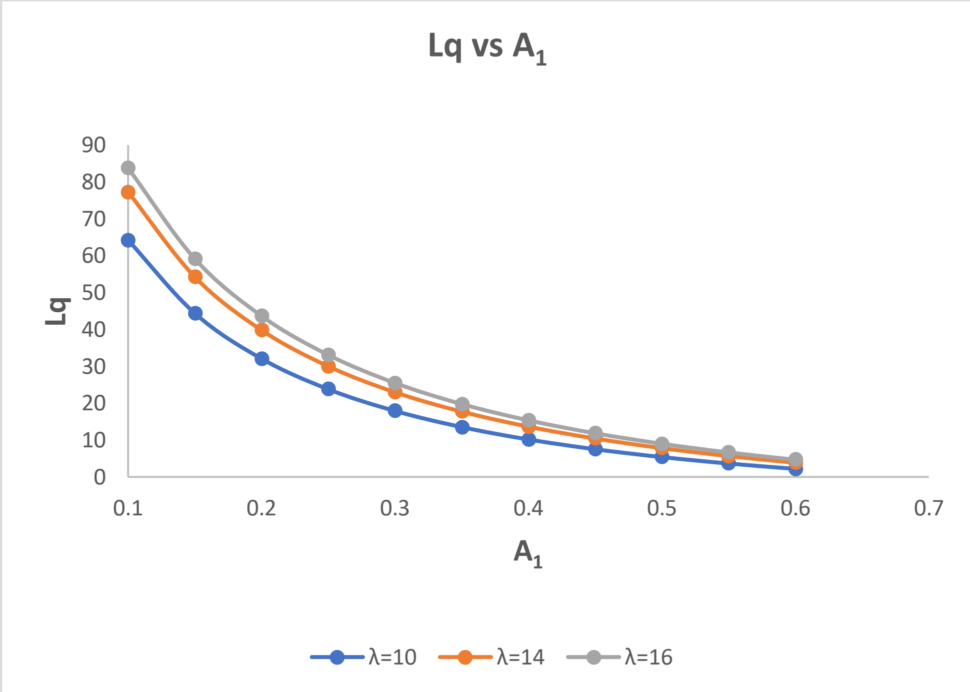

Fig. 4.2

Following can be interpreted from **Table 4.2** and **Fig. 4.2**:
(i) Mean queue length (L_q) get increased with the increase in λ.
(ii) Mean queue length (L_q) decreases with respect to the increase in probability (A_1).

4.3 Behaviour of mean queue length of the system (L_q) with respect to probability D_4 for different values of first server service rate (μ_1) is depicted in Table 4.3 and in Fig. 4.3 keeping the values of other parameters as fixed.

Table 4.3

$\lambda=2$, $\mu_2=4$, $\mu_3=5$, $\mu_4=0.2$, $A_1=0.8$, $A_{12}=0.2$, $B_2=0.5$, $B_{21}=0.2$, $B_{23}=0.3$, $C_3=0.6$, $C_{32}=0.15$, $C_{31}=0.2$, $C_{34}=0.05$, $D_{42}=0.15$, $D_{43}=0.05$			
	μ_1=1	**μ_1=1.5**	**μ_1=2**
D_4	**Lq**	**Lq**	**Lq**
0.1	6.398197	5.840152	4.649873
0.15	6.392146	5.827326	4.62022
0.2	6.38604	5.814352	4.590124
0.25	6.379878	5.801228	4.559576
0.3	6.373659	5.787952	4.528565
0.35	6.367384	5.77452	4.497084
0.4	6.36105	5.760932	4.465121
0.45	6.354658	5.747183	4.432667
0.5	6.348206	5.733272	4.399711
0.55	6.341694	5.719195	4.366243
0.6	6.335121	5.704951	4.332253

REDVET - Revista electrónica de Veterinaria - ISSN 1695-7504
Vol 25, No. 2 (2024)
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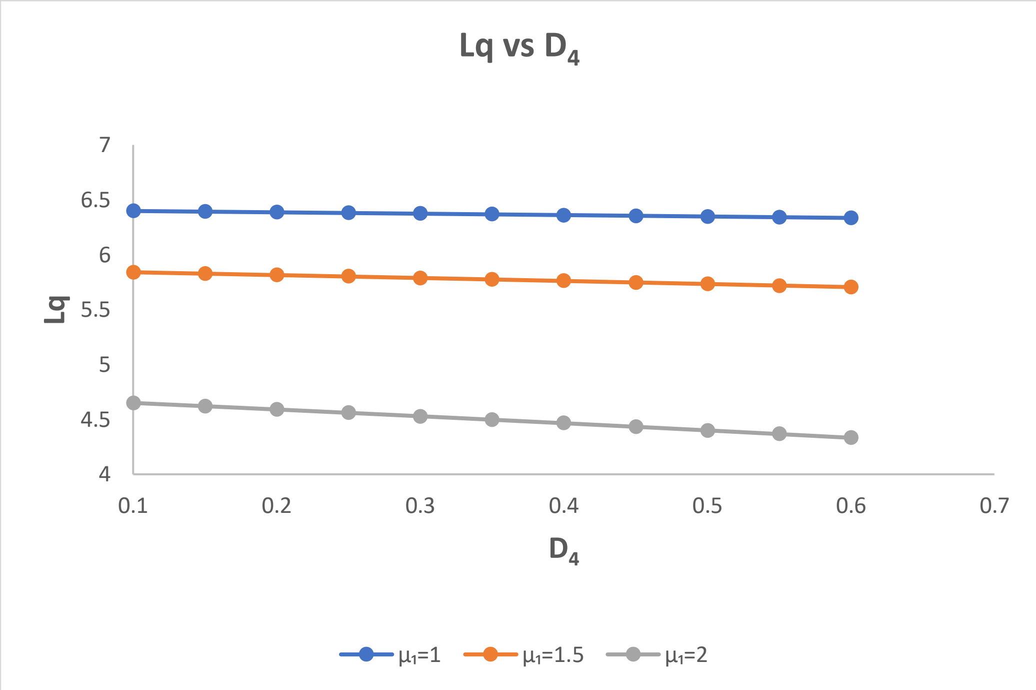

Fig. 4.3

Following can be interpreted from **Table 4.3** and **Fig. 4.3**:
(i) Mean queue length (L_q) get decreased with the increase in D_4.
(ii) Mean queue length (L_q) decreases with respect to the increase in first server service rate (μ_1).

4.4. Behaviour of mean queue length of the system (L_q) with respect to probability D_{41} for different values of first server service rate (μ_1) is depicted in Table 4.4 and in Fig. 4.4 keeping the values of other parameters as fixed.

Table 4.4

$\lambda=2$, $\mu_2=4$, $\mu_3=5$, $\mu_4=0.2$, $A_1=0.8$, $A_{12}=0.2$, $B_2=0.5$, $B_{21}=0.2$, $B_{23}=0.3$, $C_3=0.6$, $C_{32}=0.15$, $C_{31}=0.2$, $C_{34}=0.05$, $D_{42}=0.15$, $D_{43}=0.05$			
	μ_1=1	**μ_1=1.5**	**μ_1=2**
D_{41}	**Lq**	**Lq**	**Lq**
0.1	6.321788	5.675947	4.262656
0.15	6.328486	5.690536	4.297727
0.2	6.335121	5.704951	4.332253
0.25	6.341694	5.719195	4.366243
0.3	6.348206	5.733272	4.399711
0.35	6.354658	5.747183	4.432667
0.4	6.36105	5.760932	4.465121
0.45	6.367384	5.77452	4.497084
0.5	6.373659	5.787952	4.528565
0.55	6.379878	5.801228	4.559576
0.6	6.38604	5.814352	4.590124

REDVET - Revista electrónica de Veterinaria - ISSN 1695-7504
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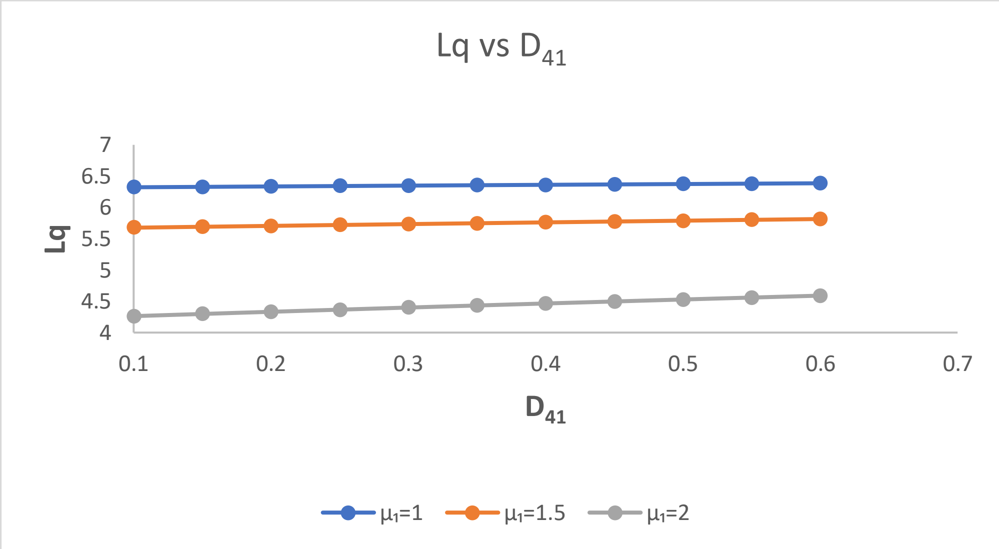

Fig. 4.4

Following can be interpreted from **Table 4.4** and **Fig. 4.4**:
(i) Mean queue length (L_q) increases with the increase in D_{41}.
(ii) Mean queue length (L_q) decreases with respect to the increase in first server service rate (μ_1).

4.5. Behaviour of mean queue length of the system (L_q) with respect to arrival rate (λ) for different values of first server's service rate (μ_1) is depicted in Table 4.5 and in Fig. 4.5 keeping the values of other parameters as fixed.
Table 4.5

μ_2 = 4, μ_3= 5, μ_4= 0.2, A_1=0.8, A_{12}= 0.2, B_2=0.5, B_{21}=0.2, B_{23}=0.3, C_3=0.6, C_{32}=0.15, C_{31}=0.2, C_{34}=0.05, D_4=0.5, D_{41}=0.3, D_{42}=0.15, D_{43}=0.05			
	μ_1=5	**μ_1=6**	**μ_1=7**
λ	**Lq**	**Lq**	**Lq**
10	5.794329	4.784608	2.705374
11	6.239058	5.637136	4.433117
12	6.581448	6.250897	5.575211
13	6.856482	6.718201	6.389959
14	7.084974	7.089733	7.005123
15	7.280035	7.39546	7.490465
16	7.450314	7.654158	7.887031
17	7.601737	7.878152	8.220421
18	7.738493	8.075849	8.507372
19	7.863625	8.253165	8.759255
20	7.979393	8.414383	8.984049

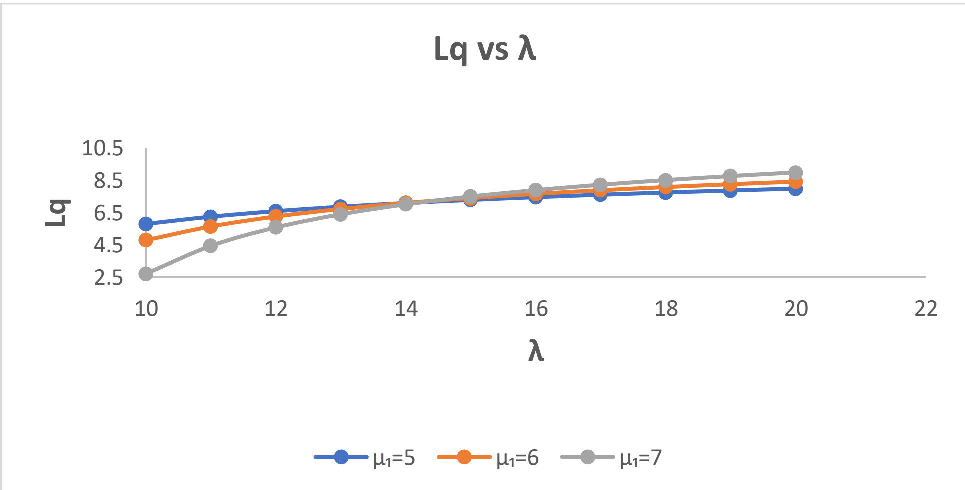

Fig. 4.5

REDVET - Revista electrónica de Veterinaria - ISSN 1695-7504
Vol 25, No. 2 (2024)
http://www.veterinaria.org
Article Received: Revised: Accepted:

Following can be interpreted from **Table 4.5** and **Fig. 4.5**:
(i) Mean queue length (L_q) increases with the increase in λ.
(ii) Mean queue length (L_q) decreases with respect to the increase in first server service rate (μ_1) for λ<14 but increases for λ≥14.

4.6. Behaviour of mean queue length of the system (L_q) with respect to service rate of first server (μ_1) for different values of service rate of second server (μ_2) is depicted in Table 4.6 and in Fig. 4.6 keeping the values of other parameters as fixed.

Table 4.6

$\lambda=10$, $\mu_3=5$, $\mu_4=0.2$, $A_1=0.8$, $A_{12}=0.2$, $B_2=0.5$, $B_{21}=0.2$, $B_{23}=0.3$, $C_3=0.6$, $C_{32}=0.15$, $C_{31}=0.2$, $C_{34}=0.05$, $D_4=0.5$, $D_{41}=0.3$, $D_{42}=0.15$, $D_{43}=0.05$			
	$\mu_2=3$	**$\mu_2=4$**	**$\mu_2=5$**
μ_1	**Lq**	**Lq**	**Lq**
1	8.074372	6.640156	5.572566
1.5	8.23663	6.639927	5.507467
2	8.415145	6.625059	5.418744
2.5	8.61269	6.590746	5.30008
3	8.832768	6.530516	5.143193
3.5	9.079882	6.435567	4.937096
4	9.359947	6.293756	4.66701
4.5	9.680937	6.088082	4.312751
5	10.05392	5.794329	3.846272
5.5	10.49483	5.377296	3.227816
6	11.02758	4.784608	2.399714

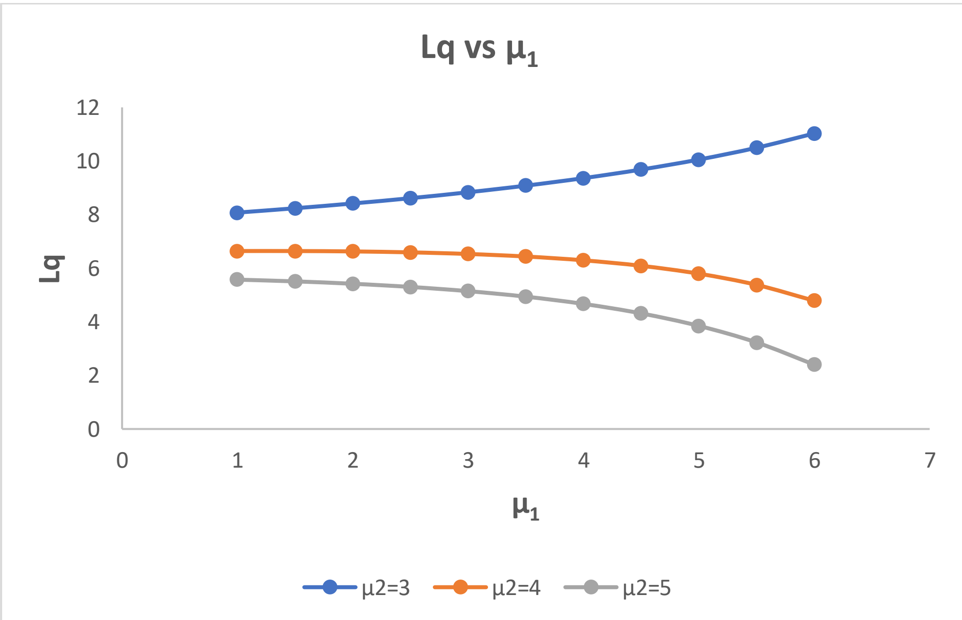

Fig. 4.6

Following can be interpreted from **Table 4.6** and **Fig. 4.6**:
(i) Mean queue length (L_q) decreases with the increase in μ_2.
(ii) Mean queue length (L_q) increses upto **μ_2<4** and decreases for **μ_2≥4,** with respect to the increase in first server service rate (μ_1).

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Vol 25, No. 2 (2024)
http://www.veterinaria.org
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4.7. Behaviour of mean queue length of the system (L_q) with respect to service rate of first server (μ_1) for different values of service rate of third server (μ_3) is depicted in Table 4.7 and in Fig. 4.7 keeping the values of other parameters as fixed.

Table 4.7

$\lambda=10$, $\mu_2=3$, $\mu_4=0.2$, $A_1=0.8$, $A_{12}=0.2$, $B_2=0.5$, $B_{21}=0.2$, $B_{23}=0.3$, $C_3=0.6$, $C_{32}=0.15$, $C_{31}=0.2$, $C_{34}=0.05$, $D_4=0.5$, $D_{41}=0.3$, $D_{42}=0.15$, $D_{43}=0.05$			
	$\mu_3=4.5$	$\mu_3=5$	$\mu_3=5.5$
μ_1	Lq	Lq	Lq
1	15.01862	8.074372	6.011583
1.5	15.14726	8.23663	6.212155
2	15.2854	8.415145	6.436979
2.5	15.43394	8.61269	6.691135
3	15.59384	8.832768	6.981291
3.5	15.76607	9.079882	7.316397
4	15.95155	9.359947	7.708747
4.5	16.15102	9.680937	8.175702
5	16.36483	10.05392	8.742557
5.5	16.59241	10.49483	9.447534
6	16.83156	11.02758	10.3509

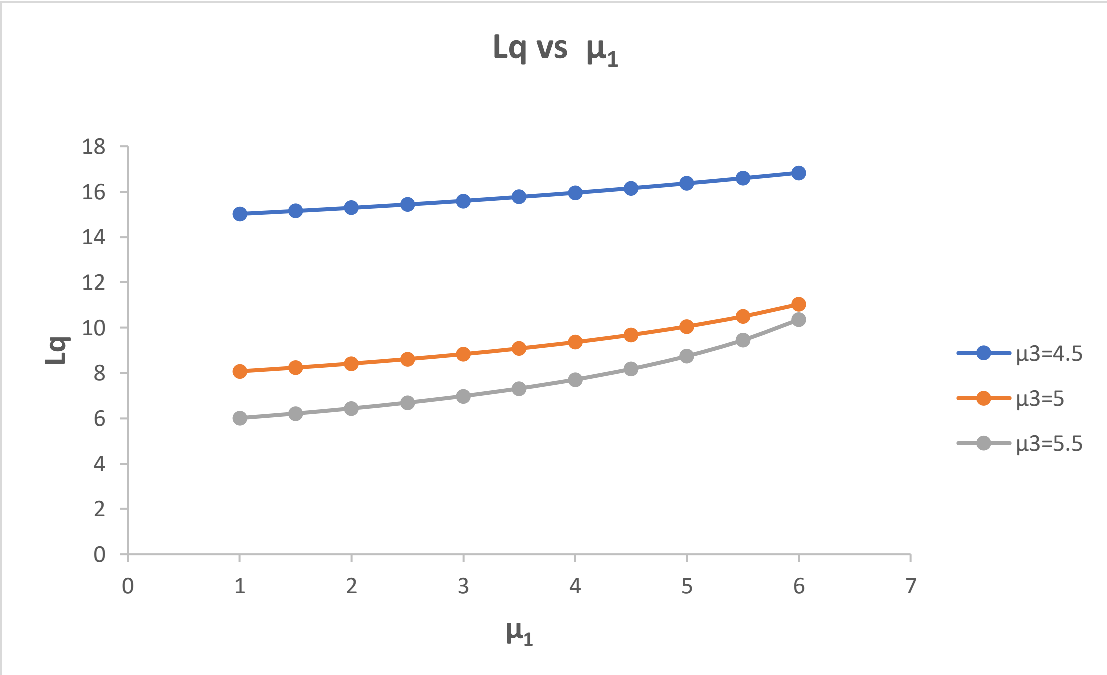

Fig. 4.7

Following can be interpreted from **Table 4.7** and **Fig. 4.7**:
(i) Mean queue length (L_q) decreases with the increase in μ_3.
(ii) Mean queue length (L_q) increses with respect to the increase in first server service rate (μ_1).

4.8. Behaviour of mean queue length of the system (L_q) with respect to service rate of first server (μ_1) for different values of service rate of fourth server (μ_4) is depicted in Table 4.8 and in Fig. 4.8 keeping the values of other parameters as fixed.

Table 4.8

λ=10, μ_2= 3, μ_3= 4, A_1=0.8, A_{12}= 0.2, B_2=0.5, B_{21}=0.2, B_{23}=0.3, C_3=0.6, C_{32}=0.15, C_{31}=0.2, C_{34}=0.05, D_4=0.5, D_{41}=0.3, D_{42}=0.15, D_{43}=0.05			
	μ_4=4.5	**μ_4=5**	**μ_4=5.5**
μ_1	**Lq**	**Lq**	**Lq**
1	1.38047	2.128096	2.998237
1.5	2.081013	2.979197	4.035891
2	2.929079	4.02242	5.325013
2.5	3.972706	5.324533	6.959272
3	5.281691	6.984686	9.081302
3.5	6.960685	9.15544	11.91728
4	9.172531	12.08172	15.84395
4.5	12.18219	16.17766	21.52846
5	16.44496	22.19033	30.24831
5.5	22.79901	31.58488	44.71771
6	32.93149	47.57593	71.60594

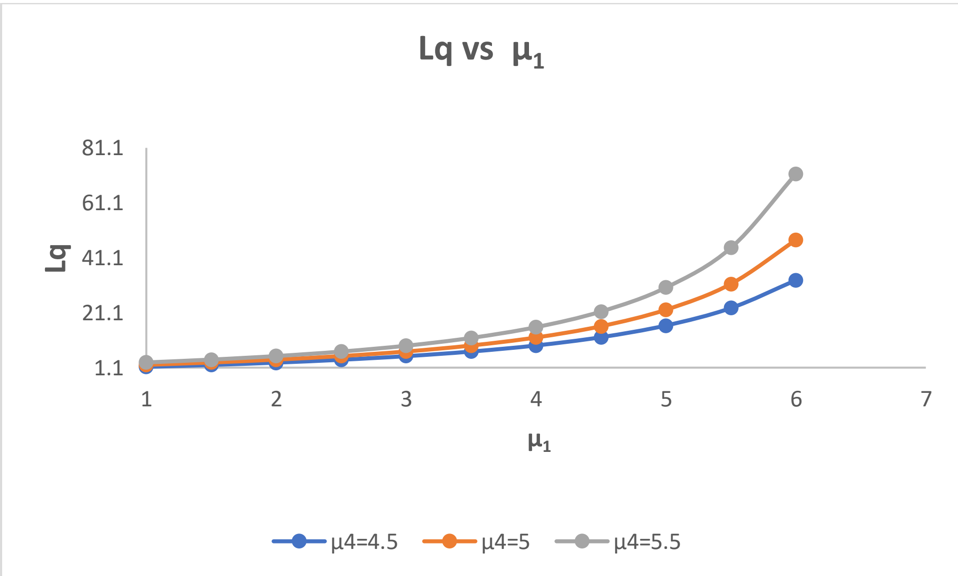

Fig. 4.8

Following can be interpreted from **Table 4.8** and **Fig. 4.8**:
(i) Mean queue length (L_q) increases with the increase in μ_4.
(ii) Mean queue length (L_q) increses with respect to the increase in first server service rate (μ_1).

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