REDVET - Revista electrónica de Veterinaria - ISSN 1695-7504
Vol 25, No.1 (2024)
http://www.veterinaria.org
Article Received:10 May 2024 Article Revised: 4 June 2024 Article Accepted: 15 July 2024

A Survey Concerning Application Of Certain Graph Labelling

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1.1. Introduction
Graph theory is a crucial area in many branches of science and engineering. One of the main areas of study in graph theory is graph labelling. Many fields, including coding theory, radar, astronomy, circuit design, missile guidance, communication network addressing, x-ray crystallography, data mining, image processing, cryptography, software testing, information security, and communication networks, among others, employ graph labelling. Future developments in cloud computing, signal processing, etc. should be used to graph labelling. Most of the graph labeling methods originated from graceful labeling by Rosa in 1967[7].

1.2. Graph Labelings
A labeling of a graph is a function defined from graph elements (vertices, edges and faces) to number. The most used graph elements for labeling are vertices and edges. If the domain of the function is a vertex set, then it is called vertex labeling. If the domain of the function is an edge set, then it is called edge labeling. If the domain of the function is the union of vertex set and edge set, then it is called total labeling[4]. The study of labeling is mainly focus on the generation of the weights on the induced labels by using well defined binary operation that satisfies certain conditions. For instance, $Weight(uv) = weight(u) * Weight(v)$, Where * is a binary operation and uv is an edge in a graph. There are many engineering problems that can be solved by using suitable graph labeling.

1.3. Application of Radio labeling in sensors
Every station point is given a channel by a series of transmitters so that interference can be prevented. The stronger the distance between interference, the smaller the distance between stations; consequently, the difference in channel assignment grows. Consider every vertex to be a transmitter and every pair of vertices that are joined by an edge to be a transmitter that is neighboring. Minimizing distance can be solved by the type of labeling called Radio labeling, that is introduced by Gray Chartrand in 2001. It is defined as the labelling f of nonnegative integers to the vertices such that $|f(u) - f(v)| \geq diam(G) + 1 - d(u, v), \forall u, v \in V(G)$, where d(u,v) is the distance between the vertices (u,v) and diam(G) is the diameter(maximum distance between any pair of vertices) of G [1].

1.4. Application of magic labeling in routing
Any conventional network can be depicted as a graph structure, such as a wheel, fan graph, cycle, or friend graph, but it must be fixed first. Once labeling is applied, the network may automatically identify routes with any extra information. Magic labeling is a viable solution to the aforementioned issue, as it produces a magic constant that is accessible to the network. The magic constant, the router's own label, and the labels supplied to the channels are now used by the router to automatically identify the next node to be reached. Searching the next node in routing can be solved by magic labeling[11]. Magic labeling has its origin from magic squares. In 1963, Sedlacek introduced the magic valuation(labeling) which is defined as the assignment of labels to the edges such that the weights(the total of the labels of the edges that align with the vertex) induced on vertices are constant, that is called magic constant[8].

1.5. Application of edge magic total labeling in addressing systems
Determination of efficient addressing systems can be solved by using edge-magic total labelling. Suppose the communication network serves as a representation of graph G. If edge magic total labeling f exists for G, having magic constant k, assign the labels of the vertices and edges to nodes and links respectively. Furthermore, a distinct address is established for communications sent by the system operator to the nodes; the address $f(v)$ is possessed by the link that connects the system to node [3]. Suppose edge-magic total labeling does not exist for the graph structure, edge-magic injection can be used. The set of all vertices and edges is the domain of an edge magic total labeling, also known as a total labeling. Edge magic labeling was first described by Ringel and Llado in 1996. It is the process of labeling an edge so that, regardless of the edge selected, the total of all labels associated with it equals a constant. [6]

1.6. Application of graceful labeling in addressing systems
The challenge lies in allocating addresses to potential links within a communication network. It is necessary for each address to be unique and for the address of a link to be inferred from the identities of the two linked nodes, avoiding the need to keep data in the form of a table. It can be solved by finding the difference between the two addresses[3]. For this situation, graceful labelling can be applied. In 1967, Alexander Rosa invented the graceful labeling which is

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defined as the injective mapping from vertices to 1,2,q so that $|f(u) - f(v)|$ are all distinct concerning any pair of vertices u and v, where q is the quantity of edges of graph[7]. Edge magic labelling can also be used to solve the problem of addressing the system in a network.

1.7. Application of odd graceful labeling in crystallography
One of the most effective methods for determining the structural characteristics of crystalline solids is X-ray diffraction, which involves striking a crystal with an X-ray beam and diffracting it in a variety of directions. There are instances where many structures provide identical diffraction information. In terms of mathematics, this problem is comparable to figuring out how to label every acceptable graph that results in a predetermined set of edge labels [13]. This can be solved by odd graceful labeling. If it is feasible to label the vertices of a connected graph G with pairwise distinct numbers in {0, 1, 2, 3,......, 2q - 1} so that each edge, uv, is labeled $|f(u) - f(v)|$, and the resultant edge labels are the whole set {1, 3, 5, 2q - 1}, then the graph is said to be odd graceful [5].

1.8. Application of semi graceful labeling in radars
Radar pulse codes are generated using semi-graceful labeling. The problem is to time the pulses such that it has low amplitude and more energy. Sending out a pulse or series of pulses and then waiting for the return signal after reflection constitutes the way radar distance is measured. Distributed like a transmission pulse sequence comprises an array of detectors. [3]. The problem of finding the sequence can be solved by semi graceful labeling, which is originated from graceful labeling.

1.9. Application of mean labeling in cryptography
Mean labeling can be used to raise the Affine Cipher's security level while encrypting text on social media. The study of this problem is carried out by Sudarsana et.al in 2020[12]. By using this labeling, Affine Cipher's ability to encrypt a text on social networking platforms with two levels of security is strengthened. In particular, SU(Super mean) labeling is applied to encrypt the text. So graph labeling plays a vital role in cryptography. In 2003, Somasundaram and Ponraj coined the mean labeling f which is defined as the injective mapping from vertices to 0,1,2 q so the induced edge labels are distinct, that is $\frac{f(u)+f(v)}{2}$ If $f(u) + f(v)$ is even and f(u)+f(v)+1 if $f(u) + f(v)$ is odd, where q is the quantity of edges2 of graph. Super mean labelling is the mean labelling in which the codomain of the labelling function is the union of vertex set and edge set of graph G[9].

1.10. Application of geometric mean labeling in traffic flow
The labeling issues in graph theory can be applied to traffic studies by treating the average traffic in each street as the weight of the corresponding edge and the crowd at each intersection as the weight of a vertex. If we suppose that the predicted traffic at each street is equal to the arithmetic mean of the weight of the terminal vertices, it generates the mean labeling of the graph. The pace of expansion of traffic in each street will be more realistic when we use a geometric mean rather than an arithmetic mean in a huge city[1]. The geometric mean labeling was defined by Somasundaram et al. as follows: A graph G = (V, E) with p vertices and q edges is considered to be a geometric mean graph if it is possible to label the vertices $x \epsilon V$ with distinct labels $f(x)$ from 1,2,...,q+1 so that when each edge $e = uv$ is labelled with $f(uv) = \left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ or $\left\lceil \sqrt{f(u)f(v)} \right\rceil$ then the edge labels are distinct. , for the edge uv, they have used flooring function $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ and for the edge vw, they have used ceiling function $\left\lceil \sqrt{f(v)f(w)} \right\rceil$ for fulfilling their requirement[10].

1.11. Conclusion
Graph Labeling is a potent tool that facilitates a variety of networking and communication domains as explained above. A brief overview is provided on how to approach the topic of graph labeling. In addition to learning new concepts relevant to their area of study, researchers may receive some knowledge on graph labeling and its uses in traffic flow, addressing systems, radar sensors, routing, traffic flow, and cryptography.

References
1. Chartrand G, Erwin D, Zhang P and Harary F, Radio Labeling of Graphs, Bull, Inst. Combin. Appl., 33(2001), 77-

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85.
2. Durai Baskar A and Arockiaraj S, F-Geometric mean graphs, International Journal of Applications and Applied Mathematics, 10 (2015), 937–952.
3. Hartsfield N and Ringel G, Pearls in Graph Theory, Academic Press, San Diego, 1990
4. Kotzig A and Rosa A, Magic valuations of finite graphs, Canad. Math. Bull.,13 (1970), 451–461
5. Lo S P, On edge-graceful labelings of graphs, Congress Numer., 50 (1985), pp. 231- 241
6. Ringel G and Llado A, Another tree conjecture, Bull. Inst. Combin. Appl. 18 (1996), 83- 85.
7. Rosa A, On certain Valuations of the vertices of a graph, Theory of graphs, (International Symposium, Rome),(1967), 349–355.
8. Sedlacek J, Problem 27, Theory of graphs and its applications, Proc. Symp. Smolenice, (1963), 163–164.
9. Somasundaram S and Ponraj R, On mean graphs of order ¡ 5, Decision and Mathematical Sciences, 9 (2004) 47–58.
10. Somasundaram S, Vidhyarani P and Ponraj R, Geometric Mean Labeling of Graphs, Bulletin of pure and Applied Sciences,(Section E Maths and Statistics) 30E (2011), 153–160.
11. Stewart B M, Magic graphs, Canadian J. Math., 18 (1966), 1031– 1059
12. Sudarsana W, Suryanto S A, Lusianti D and Putri N P A P S, An application of super mean and magic graphs labeling on cryptography system, Journal of Physics: Conference series, 1763(2020), 16-17
13. Teraiya O M, Application of Graph Theory in Various Field of Applied Science & Engineering, Journal of Emerging Technologies and Innovative Research, 6(2019), Issue 3,516-520.