REDVET - Revista electrónica de Veterinaria - ISSN 1695-7504
Vol 24, No. 4 (2023)
http://www.veterinaria.org
Article Received:September 2023 Revised:October 2023 Accepted:November 2023

One-Way Tape Determinsitic PDA

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Abstract. An automata with a tape is known as a pushdown automata (PDA). A pushdown automation has an advantage of scanning the alphabets of its tape without deleting its content. A one-way tape deterministic pushdown automata (*DPDA*) is considered in this paper. In this paper, it is presented that L(R) ={w| $x \in R, wx \in L$} where R is a set of symbols and a language L accepted by a PDA. On the basis of a corollary, the terminal symbols are not required on the alphabet set of *DPDA*. In this paper it is also presented that $Max(L)$ = {w| $x \in R, wx \in L$} is accepted by a *DPDA* only and only if L is accepted by a *DPDA*.

Keywords: Automata, Deterministic, Pushdown, Quotient, Transition Functions.

Introduction

A *DPDA* has been studied as an acceptance device [1, 2]. The basic nature of PDA is nondeter- ministic consisting of two-way tape with terminal symbols and a transition function [3, 4]. The be- havior of proposed DPDA is totally different form the existing PDA [5], the DPDA [6] searches the tape in a read-only manner. In the theoretical computer science, the closure properties on the exist- ing language family has proven important. Some closure properties of deterministic PDA have been well-established for novel *DPDA* [7, 8]. The following operations have been shown closed for thelanguages accepted by the proposed novel *DPDA*.

- Kleen Closure
- Quotient with a Regular Set
- Union
- Concatenation
- Intersection

In this paper, author try to establish analogous result for the languages for novel *DPDA* which was previously unknown. The more interest in this result originate from the fact that all the existing axiomsexcept complement, have been shown closure for proposed *DPDA* [1, 2]. Furthermore, there exist some families of languages accepted by novel *DPDA* that are not closed under quotient [5]. Themethod used in this paper for proof is new and has some other applications. The elimination of terminalsymbols is one more fact to increase the existing interest in quotient (on regular set) L_1/L_2= { {w x L_2, wx L_1 where L_1 and L_2 are languages and L_1/L_2 is the quotient. For *DPDA*, it is admitted that terminal symbols on the alphabet are avoidable for the non-deterministic PDA [9].

Definition of DPDA

Formally, *DPDA* represented as $1_{W\ PDA}$ is a seven-tuple (Q_k, Σ, δ_B, δ, Q_o, Q_F , η) machine such that

1. *"Qk : a finite set of states"*
2. *"Σ : a non−empty finite set of input symbols"*
3. *"δ : is a transition from* $\delta(Qk, \Sigma \cup \varepsilon, \eta - \{B\}) \subseteq \delta(Qk, S, R, L\})$ *where range of δ is that the tape head remains stationary, moves right or moves left respectively"*
4. *"δB : is a transition from* $\delta(Qk, \Sigma \cup \varepsilon, \eta - \{B\}) \subseteq \delta(Qk, S, E, L\}) \cup (\eta - \{B\})$ *when top of the tape has blank symbol and E imply a move left followed by writing blank over rightmost non − blank; L implies a move left without erasing rightmost non − blank respectively."*
5. *"Qo : an initial state"*
6. *"QF : a finite set of final state where* $QF \subseteq Q$*"*
7. *"η : a finite set of tape symbols including B & Zo where B is blank and Zo is top of tape"*

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Configuration of Transition Function δ of $1_{W\ PDA}$

A δ has three arguments Q, $\Sigma \cup \varepsilon$ and $\eta - \{B\}$. When the second argument of δ is ε the tape head moves either static (S) or right (R) or left (L) without reading any symbol from the input. The movement of δ transition function also depends on the Br (non-blank rightmost symbol) with Qc (current state) and Σ (input symbol). The initial configuration of *DPDA* is $\delta(Qk, \Sigma \cup \varepsilon, \eta B \cup Zo)$. It has been noted that Zo is never written by *DPDA*, Zo is an indicator of left end of the tape. One important fact is presumed that the tape head never moves beyond Zo to the left i.e δ $(Qp, \Sigma \cup \varepsilon, Zo)$ does not contain
(Qq, L).

The initial configuration of transition function of 1*WPDA* is denoted by δ $(Qi, \varepsilon y, \eta i)$, where $Qi \subset Q$, $\varepsilon y \subset \Sigma \cup \varepsilon$ and ηi is an integer having value either 0 or 1. The value of ηi=0 represents that the position of head is on the top of tape and the rightmost non-blank symbol is represented by ηi=1. In this paper the configuration of transition function is defined as under:

1. $\eta_i > 0$, *if* $(Q_1, D) \in \delta(Q_k, y, Z_0) \mid y : (Q_k, x_1Z_0x_2, \eta_i) \vdash 1_{WPDA} (Q_1, x_1Z_0x_2, \eta_i + \gamma)$ *where* γ *= -1, 0 or +1 respectively as D = R, S or L.*

2. *if* $(Q_1, L) \in \delta_B(Q_k, y, Z_0) \mid y : (Q_k, xZ_0, 0) \vdash 1_{WPDA} (Q_1, xZ_0, 1)$.

3. *if* $(Q_1, E) \in \delta_B(Q_k, y, Z_0) \mid y : (Q_k, xZ_0, 0) \vdash 1_{WPDA} (Q_1, x, 0)$.

4. *if* $(Q_1, S) \in \delta_B(Q_k, y, Z_0) \mid y : (Q_k, xZ_0, 0) \vdash 1_{WPDA} (Q_1, xZ_0, 0)$.

5. *if* $(Q_1, Z_i) \in \delta_B(Q_k, y, Z_0) \mid Z_i \in \eta$ & $y : (Q_k, xZ_0, 0) \vdash 1_{WPDA} (Q_1, xZ_0Z_i, 0)$.*If for* $1 \le j \le n$, $y_j : (Q_j, x_j, \eta_j) \vdash 1_{WPDA} (Q_{j+1}, x_{j+1}, \eta_{j+1})$, *then* $\mid y_1y_2.....y_n : (Q_1, x_1, \eta_1) \vdash^* 1_{WPDA} (Q_{n+1}, x_{n+1}, \eta_{n+1})$.

So, on the basis of above description it is stated that any automata, to be a ($1_{W\ PDA}$) if, intuitivelyfrom the above given configuration there is at most one move possible. A language (L) is always accepted by a final state of the given *DPDA*. Let the language L($1_{W\ PDA}$) accepted by a $1_{W\ PDA}$, isthe set w w : $(Q_0, \eta_0, 0)$ $_1WPDA$ (Q_k, x, η_i) for some Q_k Q_F , x η and η_i for some integer i) is called a $1_{W\ PDA}$ language.

Finding

With the help of lemma the authors try to simplify proof of main theorem in this section.

Theorem 1. Let A be a *DPDA* and assume $X_1 = (Q_1, Z_1), (Q_2, Z_2), ..., (Q_m, Z_m)$ and $X_2 = (R_1, Y_1), (R_2,Y_2), ..., (R_n, Y_n)$ are sets of state instance and tape symbol.

Proof: Let $w \subseteq$ L be a language where $w \in I^*$ such that either

1. w : $(Q_0, Z_0, 0) \vdash^* 1WPDA$ $(Q_k, xZ, 0)$ for some x in η-$\{B\}^*$ and (Q_k , Z_0) in x_1, or
2. w : $(Q_0, Z_0, 0) \vdash^* 1WPDA$ (Q_k, x_1Zx_2, j) where j = $|x_2|$ + 1 and (Q_k, Z_0) in x_1.

In the above given description, authors assume $1'$W P DA is a newly constructed *DPDA* from $1_{W\ PDA}$ such that $1'$W P DA accept the language L'. The formal definition of this newly *DPDA* is as under:

$1'_{WPDA} = (Q'_k, \Sigma', \delta'_B, \delta', Q'_o, Q'_F, \eta')$ where $Q' = \{Q_0, Q'_0, Q''_0 \mid Q_0 \in Q_k\}$, $Q_F = Q' \mid Q' \in Q_k$ and δ'_B, δ' are defined as under.

1. If $\delta(Q_k, x, \eta) = (Q_1, D)$ where $D \in \{L, S, R\}$ then $\delta'(Q_k, x, \eta) = (Q''_1, D)$.
2. If $(Q_k, \eta) \in x_2$, then $\delta'(Q'', \varepsilon, \eta) = (Q', S)$ and $\delta'(Q', \varepsilon, \eta) = (Q_k, S)$.
3. If $(Q_k, \eta) \notin x_2$, then $\delta'(Q'', \varepsilon, \eta) = (Q_k, S)$.
4. If $\delta(Q_k, x, \eta) = (Q_1, D)$ where $D \in \{S, L, E\} \cup \Sigma - \{\eta - Z_0\}$ then $\delta'_B(Q_k, x, \eta) = (Q''_1, D)$.
5. If $(Q_k, \eta) \in x_1$, then $\delta'_B(Q'', \varepsilon, \eta) = (Q', S)$ and $\delta'_B(Q', \varepsilon, \eta) = (Q_k, S)$.
6. If $(Q_k, \eta) \notin x_1$, then $\delta'_B(Q'', \varepsilon, \eta) = (Q_k, S)$.

The behaviour of transition functions δ and δ_B clearly indicate that language $L(1'$DA) accepted bynovel *DPDA* is regular language including the empty tape.

Theorem 2. Let S is a finite state machine and 1_{WPDA} be a *DP DA* so ∃ a $1'_{DA} = (Q_k, \Sigma, \delta'_B\, \delta', Q_o, Q_F, \eta C, [Z_0 \cup _0 (\text{mapping})]$ such that

$x : (Q_0, [Z_0 \cup _0], 0) \vdash *_{1WPDA} (Q_k, [Z_0 \cup _0][Z_1 \cup _1]........................[Z_n \cup _n], i)$

i ff

$x : (Q_0, [Z_0 \cup _0], 0) \vdash *_{1WPDA} (Q_k , Z_0Z_1. . . Z_n, i)$, and for $0 \le i \le n$, $_i$ describe the mapping i.e $Z_0Z_1. . . Z$.

Proof: The behaviour of transition functions δ' and δ'_B are defined as under.

1. $\delta'(Q, \alpha, [\eta,]) = \delta(Q, \alpha, \eta)$. $1'_{DA}$ moves exactly as 1_{WPDA} if the tape head of $1'_{DA}$ is in the tapeand $1'_{DA}$ ignore the mapping.

2. $\delta_B(Q, \alpha, Z_1) = (Q_1, Z_2)$ where $Z_2 \in \eta$, then $\delta'_B(Q, \alpha, [\eta, _1]) = (Q_1 , [Z_2 , _2])$ such that yZ_2 described by the mapping function $_2$ where $y \in \eta*$.

These above transition function move clearly satisfies the theorem proof.

Theorem 3. Let 1_{WPDA} be a *DPDA* and $1'_{WPDA}$ be a newly constructed machine from 1_{WPDA} according to theorem 2. So, ∃ a $1''_{DA} = (Q'', \Sigma, \delta''_B\, \delta', Q_o, Q_F, \eta C, [Z_0 \cup _0])$ such that

1. $x : (Q_0, [Z_0 \cup \varnothing_o], 0) \vdash^*_{1''_{DA}} (Q_k, [Z_0 , \varnothing_o][Z_1, \varnothing_1]. . . [Z_n, \varnothing_n], 0)$
   $iff \quad x : (Q_0, [Z_0 \cup \varnothing_o], 0) \vdash *_{1_{WPDA}} (Q_k, Z_0Z_1 . . . Z_n, 0)$

2. $x : (Q_0, [Z_0 \cup \varnothing_o], 0) \vdash^*_{1''_{DA}} ([Q_k,m_i], [Z_0, \varnothing_o]....[Z_n, \varnothing_n], n - i + 1)$, i > 0
   $iff\ x^{i=n}_{i=0} : \vdash *_{1_{WPDA}} (Q_k, Z_0Z_1...Z_n, 0)$ where $[Z_0, m_i]$ is associated with the mapping function i.e $[Z_i, \varnothing_i]$

P roof : The behaviour of the transition functions of *DP DA*'s $1'' A$ and $1_D A'$ look like similar except the tape movement of $1'_{WPDA}$. So

1. if $\delta'_B(Q_k, \alpha, [\eta, \varnothing]) = (Q_1, S), (Q_1, E)$ or $(Q_1, [Z_1, \varnothing_1])$ then
   $\delta''_B(Q_k, \alpha, [\eta, \varnothing]) = \delta'_B(Q_k, \alpha, [\eta, \varnothing])$
2. if $\delta'_B(Q_k, \alpha, [\eta, \varnothing]) = (Q_1, L)$ then $\delta''_B(Q_k, \alpha, [\eta, \varnothing]) = [z_1, m_0], L)$
   where top of the tape point (B_r) non-blank symbol which is associated with the mapping function $m_0 \in [\eta, \varnothing]$.
3. For every Q in Q_k, let $\hat{Q} \in \hat{Q}_k$ where $\hat{Q}_k$ is a set of all new symbol $\hat{Q}$ such that
4. if $\delta'(Q_k, \alpha, [\eta, \varnothing]) = (Q_1, L)$, then each $m \in M$, $\delta''([Q_k, m], \alpha_1, [\eta, \varnothing]) = (\hat{Q}, \alpha, [\eta, \varnothing])$
5. if $\delta'_B(Q_k, \alpha, [\eta, \varnothing]) = (Q_1, L)$ then $\delta''_B(Q_k, \alpha, [\eta, \varnothing]) = [Z_1, m_0], L)$
6. For every $\hat{Q} \in \hat{Q}_k$, $m \in M$ and $[\eta, \varnothing] \in \eta \times C$,
   $\delta''([\hat{Q}_1, m], \epsilon ,[\eta, \varnothing]) = ([Q_1, \Delta\ (m_0),[\eta, \varnothing]], S)$
7. if $\delta'(Q_k, \alpha, [\eta, \varnothing]) = (Q_1, S)$, then each $m \in M$, $\delta''([Q_k, m], \alpha, [\eta, \varnothing]) = ([Q_1, m],S)$

Somehow, if inverse of the $o(m,[\eta,])$ i.e $o^{-1}(m,[\eta,])$ may be considered and if it determines $[\eta,]$ uniquely then it represents the transition $\delta'(Q_k, \alpha, [\eta,]) = (Q_1, R)$ with the right symbol of $[\eta,]$. If $o^{-1}(m,[\eta,])= \{m_1, m_2 , ...m_r \}$ where $r \ge 2$.

Main result

With the proof of $theorem$ 2 it is clearly visible that the class of language produced by 1_{WPDA} are closed under the $quotient\ with\ regular\ set$.
$Theorm$ 4. L/R is accepted by 1_{WPDA} if and only if $L \subseteq \Sigma \in \forall Q_F$ and $R \subseteq \Sigma$ is a regular set.
$Proof$: Let $\Psi \notin \epsilon$ and $R = R\Psi$. Let S be a $DPDA$ S= $(Q_S, \epsilon \cup \Psi, \delta_s, Q_0, Q_F)$ is accepting R'. On the basis of $Theorm$ 1, construct a $DPDA$ A which accepts $L\Psi$ by empty tape. By $Theorm$ 2 and $Theorm$ 3 construct $1^{\prime\prime}_{WPDA}$ from 1_{WPDA}. The two components X_1 and X_2 are defined as under:
X_1: $\{Q_k, [\eta, \varnothing]\}$ $\exists\, Q_1 \in Q_k$ such that $\hat{\delta}(Q_0, P_0, Q_1, P_1)$
X_2: $\{[Q_k,m], [\eta, \varnothing]\}$, $m(Q_k, P_0 = 1))$

1. $x \mid x : (Q_0, [Z_0, \varnothing_o], 0) \vdash^*_{1^{\prime\prime}_{DA}} (Q_k, y[\eta, \varnothing], 0\,), (Q_k,[\eta, \varnothing], 0\,) \in X_1$ or
2. $x \mid x : ([Q_0,m],[Z_0, \varnothing_o], 0) \vdash^*_{1^{\prime\prime}_{DA}} (Q_k, y_1[\eta,]y_2, j)$, j=$|y_2$+1$|$ and $([Q_k,m],[\eta, \varnothing]\,) \in X_2$.[10]

Summary

Max is an operation defined by $Max_{(L)}$ which represents an operation strongly related to quotientwith a regular set. $Max_{(L)} = \{\alpha \mid \alpha \in L$ and $\beta \notin \Sigma^* - \{\varepsilon\}$ is $\alpha\beta$ in $L\,\}$. By modifying the transition functions of *Theorm* 2 and *Theorm* 3 it can be shown that the family of languages accepted by non-deterministic non-erasing PDA [1] is closed under quotient with a regular set. With the help of this alternate proof it can be clearly concluded that the family of languages accepted by $1_{W\ PDA}$ is properly contained in the family of languages accepted by an 1_{DPDA}.

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